U.S. DEPARTMENT OF COMMERCE

COAST AND GEODETIC SURVEY

MANUAL OF HARMONIC CONSTANT REDUCTIONS

SPECIAL PUBLICATION NO. 260

U. S. DEPARTMENT OF COMMERCE
CHARLES SAWYER, Secretary
COAST AND GEODETIC SURVEY
ROBERT F. A. STUDDS, Director

Special Publication No. 260

MANUAL OF HARMONIC CONSTANT REDUCTIONS

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1952
Reprinted 1976

CONTENTS

HARMONIC CONSTANT REDUCTIONS

The purpose of these reductions is to obtain from the harmonic constants mean values of tidal quantities which depend upon the times and heights of high and low waters. The quantities usually sought are the mean and tropic high and low water lunitidal intervals and various tidal ranges and inequalities. While such quantities may be derived directly from the high and low water tabulations, the method of obtaining them from harmonic constants is usually less laborious and affords more consistent results because of the elimination of meterorological effects in the processes of the harmonic analysis. However, for the primary tide stations where the observations cover many years, the results obtained directly from the high and low water tabulations are usually preferred to those from the harmonic constants when the latter are based upon only a few years of observations, although in general the difference in the results may be negligible for practical purposes.

Age of inequalities.--The three principal inequalities in the tide are due to changes in the phase, parallax, and declination of the moon. In each case there is usually a lag of some hours between the time of the astronomical condition tending to produce the maximum inequality and the actual maximum as it occurs in nature. This lag is known as the age of the inequality and may be expressed in terms of the tidal constants.

Phase age.--The phase inequality is manifested by a variation in the range of tide which tends to increase in approaching the times of new and full moon and to decrease in approaching the quadratures of the moon. When the tide is represented by its harmonic constituents, the maximum range due solely to the phase effect occurs when M_2 and S_2 are in phase agreement. The origins to which the epochs of these two constituents are referred coincide at the times of new and full moon and their phase difference at this time is therefore measured by the difference in their epochs. The phase age is the time required for these two constituents to arrive at a phase agreement and can be obtained by dividing the difference in their epochs by the difference in their speeds, the latter being given in Table 2 of Coast and Geodetic Survey Special Publication No. 98. The hourly speed of S_2 exceeds that of M_2 by 1.016°, the reciprocal of which is 0.984. The required formula is as follows.

$$\text{Phase age (in hours)} = 0.984\ (S_2^\circ - M_2^\circ)\ .\ .\ .\ .\ .\ .\ .\ (1)$$

The above as well as the two following formulas is applicable regardless of whether local or Greenwich epochs are used in the computation.

<u>Parallax age.</u>--The parallax inequality results from changes in the moon's distance from the earth, the range of tide tending to increase as the moon approaches its perigee and to decrease as it approaches its apogee. This inequality is represented principally by constituent N_2, the origin of its epoch coinciding with that of M_2 when the moon is in perigee. The parallax age is therefore the interval required for constituents M_2 and N_2 to arrive at phase agreement. The reciprocal of the difference in their hourly speeds is 1.837, and the formula may be written-

$$\text{Parallax age (in hours)} = 1.837\ (M_2^\circ - N_2^\circ) \quad . \ . \ . \ . \ . \ . \ (2)$$

<u>Diurnal age.</u>--The diurnal inequality is manifested chiefly by a difference in the heights of the two high waters or of the two low waters of each day and is caused by the presence of a diurnal wave which is due to the declination of the tide-producing body. Both the moon and sun have a part in creating this wave but the moon's effect usually predominates. The diurnal wave varies throughout the month in amplitude and in its phase relation to the semidiurnal wave. Its effect on the tide as a whole is greatest at the time of the tropic tides when the amplitude is at a maximum but the high and low waters are usually affected unequally depending upon the phase relation of the two waves.

The diurnal wave is represented principally by constituents K_1 and O_1 which are in phase agreement when the wave attains its maximum amplitude. The epochs of these constituents have a common origin at the times of maximum declination and the diurnal age is the interval required for them to arrive at phase agreement. The reciprocal of the difference in their hourly speeds being 0.911, the formula for the age may be written-

$$\text{Diurnal age (in hours)} = 0.911\ (K_1^\circ - O_1^\circ) \quad . \ . \ . \ . \ . \ . \ (3)$$

<u>Mean lunitidal intervals.</u>--In the normal semidiurnal tide, the high water lunitidal interval is the time interval between the transit of the moon over a specified meridian and the occurrence of the following high water. Similarly, the low water lunitidal interval is the elapsed time between the transit of the moon and the following low water. Originally, these intervals were reckoned from the moon's transits over the local meridian of the place of observation, but more recently the practice is being adopted in the Coast and Geodetic Survey to refer all intervals to the moon's transits over the meridian of Greenwich, such intervals being designated as <u>Greenwich Intervals</u> to distinguish them from the local intervals.

As constituent M_2 is in general the predominating element in the semidiurnal tide and its epoch is referred to practically the same origin as the lunitidal intervals, this epoch reduced to time will correspond approximately to the mean high water interval, local or Greenwich according to the meridian to which the epoch itself is referred. The approximate high water interval in hours may therefore be obtained by dividing the epoch of M_2 by its speed, or more conveniently by multiplying by the reciprocal of this speed which is 0.0345. The corresponding approximate low water interval may be obtained by multiplying ($M_2^\circ \pm 180^\circ$) by the same factor.

The times of the semidiurnal high and low waters may be accelerated or retarded by the presence of shallow water constituents, the principal ones being the harmonics M_4 and M_6. The equation of a wave including the principal lunar constituent and these two harmonics may be written-

$$y = M_2 \cos (at - M_2^\circ) + M_4 \cos (2at - M_4^\circ) + M_6 \cos (3at - M_6^\circ) \quad . \; . \; . \; . \quad (4)$$

in which $\underline{a}$ is the speed of M_2 and $\underline{t}$ is time reckoned from the same origin as the constituent epoch.

The time origin may be conveniently changed to coincide with the first maximum value of the M_2 constituent, in which case the formula may be written-

$$y = M_2 \cos at + M_4 \cos (2at + 2M_2^\circ - M_4^\circ) + M_6 \cos (3at + 3M_2^\circ - M_6^\circ) \quad . \; . \quad (5)$$

For brevity let

$$P_4 = 2M_2^\circ - M_4^\circ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

$$P_6 = 3M_2^\circ - M_6^\circ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Then P_4 and P_6 are respectively the phases of constituents M_4 and M_6 corresponding to the zero phase of M_2. The values for P_4 and P_6 will be independent of whether the constituent epochs have been referred to the local or Greenwich meridian. Substituting these symbols in equation (5)

$$y = M_2 \cos at + M_4 \cos (2at + P_4) + M_6 \cos (3at + P_6) \quad . \; . \; . \; . \quad (8)$$

Values for $\underline{at}$ which will render $\underline{y}$ a maximum or minimum must satisfy the derived equation

$$M_2 \sin at + 2M_4 \sin (2at + P_4) + 3M_6 \sin (3at + P_6) = 0 \quad . \; . \; . \quad (9)$$

Referring to equation (8), the maximum or high water of the M_2 constituent occurs when $\underline{at}$ equals 0°, and the minimum or low water when $\underline{at}$ equals 180°. Let the accelerations due to M_4 and M_6 be represented by $\underline{v}$ and $\underline{w}$ for the high and low water respectively, these accelerations being expressed in degrees of the semidiurnal constituent. Then for the maximum and minimum of the compound wave, $\underline{at}$ equals $\underline{-v}$ and $\underline{(180^\circ - w)}$, respectively. Substituting these in equation (9)

$$M_2 \sin(-v) + 2M_4 \sin(P_4-2v) + 3M_6 \sin(P_6-3v) = 0 \quad . \quad . \quad . \quad . \quad (10)$$

$$-M_2 \sin(-w) + 2M_4 \sin(P_4-2w) - 3M_6 \sin(P_6-3w) = 0 \quad . \quad . \quad . \quad . \quad (11)$$

From (10)

$$-M_2 \sin v + 2M_4 \sin P_4 \cos 2v - 2M_4 \cos P_4 \sin 2v$$
$$+ 3M_6 \sin P_6 \cos 3v - 3M_6 \cos P_6 \sin 3v = 0 \quad . \quad . \quad . \quad . \quad (12)$$

Transposing and dividing by cos v

$$\tan v = \frac{2M_4 \sin P_4 \frac{\cos 2v}{\cos v} + 3M_6 \sin P_6 \frac{\cos 3v}{\cos v}}{M_2 + 4M_4 \cos P_4 \frac{\sin 2v}{2 \sin v} + 9M_6 \cos P_6 \frac{\sin 3v}{3 \sin v}} \quad . \quad . \quad . \quad . \quad (13)$$

From (11) in a similar manner

$$\tan w = \frac{-2M_4 \sin P_4 \frac{\cos 2w}{\cos w} + 3M_6 \sin P_6 \frac{\cos 3w}{\cos w}}{M_2 - 4M_4 \cos P_4 \frac{\sin 2w}{2 \sin w} + 9M_6 \cos P_6 \frac{\sin 3w}{3 \sin w}} \quad . \quad . \quad . \quad (14)$$

When the angles $\underline{v}$ and $\underline{w}$ have small positive or negative values as is usually the case, the ratios $\frac{\cos 2v}{\cos v}$, $\frac{\cos 3v}{\cos v}$, $\frac{\sin 2v}{2 \sin v}$, $\frac{\sin 3v}{3 \sin v}$, etc. are near unity and may be taken as such without materially affecting the results. Assuming unity for these ratios, equations (13) and (14) may be reduced to the following forms–

$$\tan v = \frac{2M_4 \sin P_4 + 3M_6 \sin P_6}{M_2 + 4M_4 \cos P_4 + 9M_6 \cos P_6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\tan w = \frac{-2M_4 \sin P_4 + 3M_6 \sin P_6}{M_2 - 4M_4 \cos P_4 + 9M_6 \cos P_6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Formulas (15) and (16) have heretofore been the basis for obtaining the accelerations in the high and low waters in the computation of the mean lunitidal intervals. They may be considered as essentially correct when the angles v and w are small, say less than 10°, but when the angles are larger, the omission of the factors which were dropped from formulas (13) and (14) may have a material effect on the results.

Accelerations in M_2 due to either M_4 or M_6 acting alone may be readily calculated as follows. Taking the amplitude of M_2 as unity and letting R_4 and R_6 represent the ratios M_4/M_2 and M_6/M_2 respectively, and with other symbols as before, the equation of the compound wave consisting of constituents M_2 and M_4 may be written–

$$y = \cos at + R_4 \cos (2at + P_4) \quad \ldots\ldots\ldots\ldots (17)$$

and for the compound wave consisting of constituents M_2 and M_6 as

$$y = \cos at + R_6 \cos (3at + P_6) \quad \ldots\ldots\ldots\ldots (18)$$

the time origin in each case being taken at the M_2 maximum.

The shapes of the compound waves represented by the above equations will be affected to a large degree by both amplitude and phase relations existing between the constituents. (See graphs).

Values for at which will render (17) a maximum or minimum must satisfy the derived equation

$$\sin at + 2R_4 \sin (2at + P_4) = 0 \quad \ldots\ldots\ldots\ldots (19)$$

Let v'/a and w'/a represent the accelerations in the times of the semidiurnal high and low waters, respectively, due to the presence of the M_4 constituent. Then, since the time origin of the equation was taken at the M_2 maximum, the value of at in the derived equation will equal $-v'$ for the high water and $180°-w'$ for the low water of the compound wave. Substituting these in (19)

$$\sin (-v') + 2R_4 \sin (P_4-2v') = 0 \quad \ldots\ldots\ldots\ldots (20)$$

$$\sin (180°-w') + 2R_4 \sin (P_4-2w') = 0 \quad \ldots\ldots\ldots\ldots (21)$$

From which

$$\frac{\sin v'}{\sin (P_4-2v')} = 2R_4 \quad \ldots\ldots\ldots\ldots (22)$$

$$\frac{-\sin w'}{\sin (P_4-2w')} = 2R_4 \quad \ldots\ldots\ldots\ldots (23)$$

Although the above equations do not admit a simple direct solution for v' and w', required values may be obtained by a system of trials and approximations and table 1 has been prepared giving such values for different amplitude and phase relations. These values are expressed in degrees of the semidiurnal wave and may be converted into solar hours by the application of the factor 0.0345.

From a comparison of equations (22) and (23) it is obvious that the high water accelerations are applicable to the low waters if the phase difference is altered by 180°. When P_4 is between 0° and 180°, the high water acceleration due to M_4 is positive and the low water acceleration is negative. When P_4 is between 180° and 360°, the signs of the accelerations are reversed.

As long as the ratio R_4 does not exceed 0.25, the compound wave will remain semidiurnal with a single maximum and single minimum during the semidiurnal period. When R_4 is greater than 0.25, a secondary maximum and minimum may appear with certain phase relations; and when R_4 exceeds 0.5, the compound wave becomes quarter-diurnal for all phase relations. Critical values for <u>at</u> marking the first appearance of the secondary maxima and minima must satisfy not only the first derived equation (19) but also the second derived equation-

$$\cos at + 4R_4 \cos (2at + P_4) = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

From (19) and (24)

$$\tan at = 1/2 \tan (2at + P_4) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

From which

$$\tan^3 (at + \tfrac{1}{2}P_4) = -\tan \tfrac{1}{2}P_4 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (26)$$

Critical values of <u>at</u> corresponding to different phase relations may be obtained from equation (26), and when substituted in either (19) or (24) will give the corresponding critical values for R_4. Such values have been compiled in table 3.

In computing the accelerations for table 1, no attention was usually given to the secondary maxima and minima which are in general of less amplitude than the principal maximum and minimum. However, when the phase difference is 180° the time of the semidiurnal high water remains unchanged as long as the amplitude ratio does not exceed 0.25; but when the ratio is greater than this, the high water is replaced by a secondary low water flanked by two high waters of equal height, one being considered an acceleration and the other a retardation in the original semidiurnal high water. Thus in table 1, the accelerations corresponding to the phase difference of 180° may be considered as either positive or negative.

For the compound wave (18) consisting of the constituents M_2 and M_6, the first derived equation for maxima and minima is

$$\sin at + 3R_6 \sin (3at + P_6) = 0 \quad \ldots\ldots\ldots\ldots \quad (27)$$

Letting v" and w" (expressed in degrees of the semidiurnal wave) represent respectively the high and low water accelerations due to M_6, and substituting in (27), we have

$$\sin (-v") + 3R_6 \sin (P_6 - 3v") = 0 \quad \ldots\ldots\ldots\ldots \quad (28)$$

$$\sin (180° - w") + 3R_6 \sin (P_6 + 180° - 3w") = 0. \quad \ldots \quad (29)$$

From which

$$\frac{\sin v"}{\sin (P_6 - 3v")} = 3R_6 \quad \ldots\ldots\ldots\ldots \quad (30)$$

$$\frac{\sin w"}{\sin (P_6 - 3w")} = 3R_6 \quad \ldots\ldots\ldots\ldots \quad (31)$$

From a comparison of (30) and (31), it is apparent that the accelerations due to M_6 for any phase difference are the same for both high and low waters. Table 2 contains the accelerations based upon the above equations. They are expressed in degrees of the semidiurnal wave and are to be multiplied by the factor 0.0345 to reduce to solar hours.

As long as the ratio R_6 does not exceed 1/9, the compound wave will remain semidiurnal for all phase relations, but when the ratio does exceed this amount, two secondary maxima and two secondary minima may appear with certain phase relations, and when the amplitude ratio exceeds 1/3, the compound wave will be sixth-diurnal for all phase relations. Critical values for <u>at</u> marking the first appearance of the secondary maxima and minima must satisfy both the first derived equation (27) and also the second derived equation

$$\cos at + 9R_6 \cos (3at + P_6) = 0 \quad \ldots\ldots\ldots \quad (32)$$

Therefore

$$\tan at = 1/3 \tan (3at + P_6) \quad \ldots\ldots\ldots \quad (33)$$

From (33), two critical values for <u>at</u> differing by 180° may be obtained for each value of P_6, and the corresponding value for R_6 then obtained from either (27) or (32). Critical values corresponding to the principal phase relations are included in table 3. In computing the accelerations in table 2, no attention was usually given to the secondary maxima and minima, but when the phase difference is exactly 180° and the amplitude ratio greater than 1/9, there will occur two high waters of equal height and two low waters of equal height, one of these high waters and one of the low waters being accelerations and the others retardations from the normal semidiurnal high and low waters. Thus the accelerations in table 2 corresponding to a phase difference of 180° may be considered as either positive or negative.

The combined effect of the M_4 and M_6 constituents acting together may not be the same as the sum of their individual effects when acting independently, largely because of the sensitiveness of the results to the phase relations. However, in the application of tables 1 and 2, approximate values can be obtained by first determining from table 1 accelerations $\underline{v'}$ and $\underline{w'}$ due to M_4 alone, and then taking the phase relation of M_6 to the combined (M_2+M_4) wave as (P_6-3v') for the high water and (P_6-3w') for the low water, using these arguments in table 2. The values for $\underline{v''}$ and $\underline{w''}$ thus found are then combined with $\underline{v'}$ and $\underline{w'}$ respectively to obtain accelerations $\underline{v}$ and $\underline{w}$ representing the combined effect of M_4 and M_6. The accelerations obtained by means of tables 1 and 2 do not in general differ greatly from results derived from formulas (15) and (16), but on the whole appear to be more satisfactory. The use of the tables is therefore recommended.

The accelerations $\underline{v}$ and $\underline{w}$, expressed in degrees of the semidiurnal wave, having been determined, the mean lunitidal intervals may be obtained from the following formulas-

$$\text{HWI (in hours)} = (M_2^\circ - v) \times 0.0345 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (34)$$

$$\text{LWI (in hours)} = (M_2^\circ \pm 180^\circ - w) \times 0.0345 \quad . \; . \; . \quad (35)$$

These intervals will be either Greenwich or local according to whether the M_2 epoch is referred to the Greenwich or local meridian.

<u>Mean range of tide</u>.-In the normal semidiurnal tide in which M_2 is the predominating constituent, the approximate mean range equals twice the amplitude of this constituent, or $2M_2$. The true range is a little larger due to the effects of the other constituents including both the harmonics of M_2 and constituents with incommensurable speeds.

Taking the accelerations in the times of high and low water as $\underline{v}$ and $\underline{w}$ respectively, the range of tide due to M_2 and its harmonics M_4 and M_6 may be expressed by the formula-

$$M_2(\cos v + \cos w) + M_4 \cos (P_4-2v) - M_4 \cos (P_4-2w)$$
$$+ M_6 \cos (P_6-3v) + M_6 \cos (P_6-3w)$$

$$= M_2(\cos v + \cos w)$$
$$+ M_4(\cos 2v - \cos 2w) \cos P_4 + M_4 (\sin 2v - \sin 2w) \sin P_4$$
$$+ M_6(\cos 3v + \cos 3w) \cos P_6 + M_6 (\sin 3v + \sin 3w) \sin P_6 \ldots \quad (36)$$

$$= M_2(\cos v + \cos w)$$
$$+ M_4[\cos (P_4-2v) - \cos (P_4-2w)]$$
$$+ M_6[\cos (P_6-3v) + \cos (P_6-3w)] \ldots\ldots \quad (37)$$

The increment to the mean range of tide due to constituents with speeds incommensurable with that of M_2 may be expressed by the following formula which is based upon a formula given in Harris's Manual of Tides, Part III, page 133, for the average value of the maxima of a compound wave consisting of a predominating constituent together with a number of smaller constituents. The increment to the range of tide is twice as great as that for the amplitude. Let the amplitude and speed of the predominating constituent be represented by $\underline{M_2}$ and $\underline{m_2}$ respectively, and the amplitudes and speeds of the smaller constituents by the general designations $\underline{B}$ and $\underline{b}$. The formula for the increment may then be written-

$$\tfrac{1}{2}M_2 \Sigma(Bb)^2/(M_2m_2)^2 \ldots\ldots \quad (38)$$

In the application of the above formula, it is convenient to place the constituents into three groups,- the lunar semidiurnals, the solar semidiurnals, and the diurnals, because in general the amplitude relations of the constituents in each of the groups approximate to the theoretical relations of their mean coefficients. For the first group the amplitudes are expressed in their relation to M_2, for the second group in their relation to S_2, and for the third group in their relation to (K_1+O_1).

For the purpose of the above grouping formula (38) may be written

$$\tfrac{1}{2}M_2 \Sigma[(Bb)/(M_2m_2)]^2 + \tfrac{1}{2}M_2(S_2/M_2)^2 \Sigma[(Bb)/(S_2m_2)]^2$$
$$+\tfrac{1}{2}M_2[(K_1+O_1)/M_2]^2 \Sigma[(Bb)/(K_1+O_1)m_2]^2 \ldots\ldots \quad (39)$$

Using the constituent speeds and coefficients given in Table 2 of Special Publication No. 98, the following numerical values are obtained for the summations indicated. Although K_2 is partly lunar and partly solar, it is placed in the solar group as its amplitude is usually inferred from S_2.

1st group		2nd group		3rd group	
N_2	0.0361	S_2	1.0713	K_1	0.0920
Nu	.0014	K_2	0.0797	O_1	.0400
L_2	.0008	T_2	.0037	P_1	.0099
2N	.0006	R_2	.0001	Q_1	.0014
Mu	.0005	Sum	1.1548	M_1	.0003
Lambda	.0001			J_1	.0003
Sum	.0395			OO	.0001
				Rho	.0001
				2Q	.0000
				Sum	0.1441

Substituting in formula (39) the above numerical values, we have for the increment to the mean range due to the constituents of incommensurable speeds-

$$0.020\ M_2 + 0.577\ (S_2/M_2)^2\ M_2 + 0.072\ [(K_1+O_1)/M_2]^2\ M_2$$

$$= M_2\ (\text{Table 4} + \text{Table 5}) \ldots\ldots\ldots\ldots (40)$$

in which $\text{Table 4} = 0.020 + 0.577\ (S_2/M_2)^2 \ldots\ldots\ldots\ldots (41)$

and $\text{Table 5} = 0.072\ [(K_1+O_1)/M_2]^2 \ldots\ldots\ldots\ldots (42)$

When computing the mean range of tide from the harmonic constants it has been the practice to include the empirical factor 1.02 to take account of nonpredictable inequalities. Combining (37) and (40) and including the empirical factor, we have for the mean range of tide.

$$\begin{aligned} Mn = {} & 1.02\ (\cos v + \cos w + \text{Table 4} + \text{Table 5})\ M_2 \\ & + 1.02\ M_4[\cos\ (P_4-2v) - \cos\ (P_4-2w)] \\ & + 1.02\ M_6[\cos\ (P_6-3v) + \cos\ (P_6-3w)] \ldots\ldots\ldots (43) \end{aligned}$$

Spring and neap range of tide.- Spring tides occur technically when constituents M_2 and S_2 are in phase agreement, or approximately so since this agreement exists for only an instant while consecutive high and low waters occur several hours apart. Similarly, neap tides occur when M_2 and S_2 differ in phase by 180°. Except for relatively small inequalities, the equilibrium arguments of these constituents coincide at the times of new and full moon but the constituents themselves do not conspire until some hours later because of the phase lags.

Constituent Mu_2 also has a contributing effect on the spring and neap tides. Its speed is less than that of M_2 by the same amount that the latter is less than the speed of S_2, and its equilibrium argument coincides with that of S_2 at the times of the new and full moon and also at the quadratures. At the time of spring tides when the phases of M_2 and S_2 are in agreement and also at the time of neap tides when these phases differ by 180°, the phase of Mu_2 will differ from that of S_2 by an amount equal to $(2M_2^\circ - S_2^\circ - Mu_2^\circ)$. The amplitude of the wave formed by constituents S_2 and Mu_2 at the times of spring and neap tides is therefore equal to $[S_2 + Mu_2\cos(2M_2^\circ - S_2^\circ - Mu_2^\circ)]$. The increase in range at the spring tides and the decrease at the neap tides is approximately twice the amplitude of this wave.

The formulas used for obtaining the spring and neap ranges of tide are based upon the discussion in Harris's Manual of Tides, page 144 in Part III supplemented by a footnote on page 326 of Part I. In deriving these formulas the residual effect of S_2 as represented by the expression $\frac{1}{2}(s_2/m_2)^2(S_2^2/M_2)$ or its equivalent $0.536\ S_2^2/M_2$ is first excluded from the mean range of tide. To allow for other perturbations the amplitude of the $[S_2+Mu_2\cos(2M_2^\circ-S_2^\circ-Mu_2^\circ)]$ wave is diminished by $(0.02 + 0.04[(K_1+O_1)/M_2]^2)$. The completed formulas follow-

$$Sg = Mn - 0.536\ S_2^2/M_2$$

$$+[S_2+Mu_2\cos(2M_2^\circ-S_2^\circ-Mu_2^\circ)]\times[1.96-0.08\ (K_1+O_1)^2/M_2^2] \quad . \ . \ . \ (44)$$

$$Np = Mn - 0.536\ S_2^2/M_2$$

$$-[S_2+Mu_2\cos(2M_2^\circ-S_2^\circ-Mu_2^\circ)]\times[1.96-0.08(K_1+O_1)^2/M_2^2] \quad . \ . \ . \ (45)$$

Although the mean, spring, and neap ranges derived directly from constants from different series of observations at a place may differ to some extent, the ratios of the spring and neap ranges to the mean range remain fairly constant and these ratios may be adopted as standard and applied to the mean range as obtained from the most reliable source.

Perigean and apogean range of tide.- The increased range of tide due to the nearness of the moon when in perigee and the decreased range due to its greater distance when in apogee are known respectively as the perigean and apogean ranges. The theoretical relations of these ranges to the mean range of tide based upon the fact that the tide-producing force exerted by the moon varies inversely as the cube of its distance from the earth may be expressed by the following formulas-

$$Pn/Mn = 1/(1-e)^3 = 1.18 \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (46)$$

$$An/Mn = 1/(1+e)^3 = 0.85 \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (47)$$

in which e equals the eccentricity of the moon's orbit with a numerical value of 0.055.

The principal tidal constituents contributing to this inequality in the range of tide are N_2, L_2, and 2N. Disregarding small inequalities, when the moon is in perigee the equilibrium arguments of M_2, N_2 and 2N are in phase agreement while the argument of L_2 differs by 180°. When the moon is in apogee the equilibrium arguments of M_2, L_2 and 2N are in phase agreement while the argument of N_2 differs by 180°. The speed of L_2 exceeds that of M_2 by the same amount that the latter exceeds the speed of N_2, and the speed of 2N is less than that of N_2 by the same amount. Allowing for parallax age and assuming the theoretical relations of the phase lags, the phase relations of these constituents at the times of the perigean and apogean tides will be the same as that of their equilibrium arguments when the moon is in perigee and apogee.

The compound wave formed by combining the above constituents with the principal lunar constituent M_2 will then have an amplitude equal to $(M_2+N_2-L_2+2N)$ at the time of the perigean tides and equal to $(M_2-N_2+L_2+2N)$ at the time of the apogean tides. If the theoretical amplitude relations of L_2 and 2N to N_2 are used, the above expressions become $(M_2 + 0.990\ N_2)$ and $(M_2 - 0.724\ N_2)$ respectively. Assuming the relations of the perigean and apogean ranges to the mean range to be equal respectively to the relations of the above amplitudes to that of M_2, and rounding out the numerical coefficients for convenience, we have

$$Pn/Mn = 1 + N_2/M_2 \quad . . . . \quad (48)$$

$$An/Mn = 1 - 0.75\ N_2/M_2 \quad . . . . \quad (49)$$

From a number of tests made, the results obtained by means of the above formulas did not differ materially from those derived from more elaborate formulas heretofore used, and when comparisons were made with the perigean and apogean ranges derived from high and low water observations the simplified formulas in a number of cases gave slightly improved agreements.

Diurnal (K_1+O_1) wave.- The diurnal wave due to the lunar forces is represented principally by constituents K_1 and O_1, the periods of which are such that they are in phase agreement at the time of the tropic tides and in opposite phase at the time of the equatorial tides. Similarly, the sun tends to produce a diurnal wave represented by constituents K_1 and P_1 which are in phase agreement at the time of the solstices and in opposite phase at the time of the equinoxes. Constituent K_1 is common to both waves and is due partly to lunar and partly to solar forces. The amplitude of the lunar diurnal wave varies from a maximum equal to the sum of the amplitudes of K_1 and O_1 at the time of the tropic tides to a minimum equal to their difference at the time of the equatorial tides.

Comparing the equilibrium arguments of M_2, O_1, and lunar K_1, the following relations are obtained-

$$\text{Argument } K_1 = \tfrac{1}{2}(\text{Arg. } M_2) + (s - \xi - 90°) \quad \ldots \ldots \quad (50)$$

$$\text{Argument } O_1 = \tfrac{1}{2}(\text{Arg. } M_2) - (s - \xi - 90°) \quad \ldots \ldots \quad (51)$$

It is to be noted that the term $\frac{1}{2}$(Arg. M_2) in the above formulas is referred to the upper transit of the mean moon and may not be the same as one-half the numerical value of the argument M_2 when the latter, through a rejection of 360°, has been referred to the lower transit. It will also be observed that the expression ($s - \xi - 90°$) is equal to zero at the time of maximum north declination of the moon and 180° at the time of the maximum south declination.

Equations of the K_1 and O_1 tides may now be written-

$$K_1 \text{ tide} = K_1 \cos\left[\tfrac{1}{2}(\text{Arg. } M_2) + (s - \xi - 90°) - K_1^\circ\right] \quad \ldots \quad (52)$$

$$O_1 \text{ tide} = O_1 \cos\left[\tfrac{1}{2}(\text{Arg. } M_2) - (s - \xi - 90°) - O_1^\circ\right] \quad \ldots \quad (53)$$

Let

$$\phi = (s - \xi - 90°) - \tfrac{1}{2}(K_1^\circ - O_1^\circ) \quad \ldots \ldots \quad (54)$$

Then ϕ is a variable angle with a period corresponding to the tropical month and its zero corresponding to the time of the tropic tide occurring at the maximum north declination of the moon, allowance being made for the diurnal age of the tide.

Combining (54) with (52) and (53), we obtain-

$$K_1 \text{ tide} = K_1 \cos\left[\tfrac{1}{2}(\text{Arg. } M_2) - \tfrac{1}{2}(K_1^\circ + O_1^\circ) + \phi\right] \quad \ldots \ldots \quad (55)$$

$$O_1 \text{ tide} = O_1 \cos\left[\tfrac{1}{2}(\text{Arg. } M_2) - \tfrac{1}{2}(K_1^\circ + O_1^\circ) - \phi\right] \quad \ldots \ldots \quad (56)$$

For brevity, let

$$B = \tfrac{1}{2}(\text{Arg. } M_2) - \tfrac{1}{2}(K_1^\circ + O_1^\circ) \quad \ldots \ldots \quad (57)$$

$$r = O_1/K_1 \quad \ldots \ldots \quad (58)$$

Then combining (55) and (56), we have-

$$(K_1+O_1) \text{ wave} = K_1 \cos (B + \phi) + O_1 \cos (B - \phi)$$

$$= (K_1+O_1) \cos \phi \cos B - (K_1-O_1) \sin \phi \sin B$$

$$= (K_1+O_1) \frac{(1+r^2+2r \cos 2\phi)^{1/2}}{(1+r)} \cos \left[B+\tan^{-1} \frac{(1-r)}{(1+r)} \tan \phi\right]$$

$$= C\ (K_1+O_1) \cos [\tfrac{1}{2}(\text{Arg. } M_2) - \tfrac{1}{2}(K_1^\circ+O_1^\circ) + x] \quad . \ . \ . \ (59)$$

in which

$$C = \frac{(1+r^2+2r \cos 2\phi)^{1/2}}{(1+r)} = \text{Table 6} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (60)$$

$$x = \tan^{-1} \frac{(1-r)}{(1+r)} \tan \phi = \text{Table 7.} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (61)$$

In general, the amplitude of O_1 is less than that of K_1 with the ratio $\underline{r}$ not exceeding unity. However, if the amplitude of O_1 is greater than K_1 and r is taken as the reciprocal of O_1/K_1, the above formulas and tables will still be applicable except that the sign of the tabular value of x in table 7 will be reversed. Formula (59) is the equation of a diurnal wave in which the amplitude and phase lag varies throughout the tropical month. The coefficient C has a maximum value of unity when ϕ equals 0° or 180° and a minimum value of $(K_1 \sim O_1)/(K_1+O_1)$ when ϕ equals 90° or 270°. The phase lag is represented by $[\tfrac{1}{2}(K_1^\circ+O_1^\circ)-x]$ which has an initial value of $\tfrac{1}{2}(K_1^\circ+O_1^\circ)$ at the time of the tropic tides. In the foregoing expressions, the epochs K_1° and O_1° must differ by less than 180°, otherwise they must be made comparable by adding 360° to the smaller.

In order that the tropic tides corresponding to the north and south declination of the moon may be comparable, it is necessary to refer them alternately to the upper and lower transits as the moon changes from one declination to the other. For the purpose of identification, the upper transit at the north declination and the lower transit at the south declination are called "a" transits, while the lower transit at the north declination and the upper transit at the south declination are designated as "b" transits. In computing $\underline{x}$ by formula (61), it will be noted that two values differing by 180° may be obtained for each value of ϕ. By taking all values of $\underline{x}$ in the 1st or 4th quadrants, as has been done in table 7, all corresponding phase lags will be referred to the "a" transit, the reference being to the upper transit when ϕ is in the 1st or 4th quadrants and to the lower transit when ϕ is in the 2nd or 3rd quadrants. At the critical values of 90° and 270°, the reference may be to either upper or lower transit, and if at these times the ratio $\underline{r}$ is unity, the amplitude of the diurnal wave is reduced to zero.

Combination of diurnal and semidiurnal wave.- In the following discussion the speed of the diurnal wave will be assumed to be one-half that of the semidiurnal wave. If this is only approximately true, the results may still be applicable to a single cycle of the compound wave, but with an understanding that in successive cycles the phase relation between the two component waves will be continually changing.

Let t = time reckoned for convenience from a high water of the semidiurnal wave;

a and $2a$, the respective speeds of the diurnal and semidiurnal waves;

y_1 and y_2, the respective ordinates referred to mean water level.

Taking the amplitude of the semidiurnal wave as unity,

Let R = ratio of diurnal wave amplitude to that of the semidiurnal wave;

Let P = high water phase difference, this difference being the phase of the diurnal wave corresponding to the semidiurnal high water that is taken as the time origin. Since the diurnal wave period includes two semidiurnal high waters, the phase difference, expressed in degrees of the diurnal wave, must refer to the one specified.

Equations of the two waves and their combination may now be written as follows-

$$y_2 = \cos 2at \quad \ldots \ldots \ldots \ldots \ldots \quad (62)$$

$$y_1 = R \cos (at + P) \quad \ldots \ldots \ldots \ldots \quad (63)$$

$$y = y_2 + y_1 = \cos 2at + R \cos (at + P) \quad . \quad (64)$$

By plotting equation (64) with different values for R and P, different types of tide may be illustrated. When R is small, the tide is distinctly semidiurnal with very little diurnal inequality in either high or low water heights. As R increases, the diurnal inequality increases, this inequality being unequally manifested in the high and low waters depending upon the phase difference P. The increase in the diurnal inequality is marked by a lowering of the lower high water and a raising of the higher low water until these two tides finally merge together and disappear, the tide then becoming diurnal.

The times of the high and low waters of the compound wave represented by equation (64) must satisfy its first derivative when equated to zero, thus

$$2 \sin 2at + R \sin (at+P) = 0 \quad \ldots \ldots \ldots \ldots \ldots \quad (65)$$

982152 O - 52 - 3

The times when the lower high water and higher low water merge must satisfy also the second derivative equated to zero, thus

$$4 \cos 2at + R \cos (at+P) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (66)$$

From (65) and (66)

$$\tfrac{1}{2} \tan 2at = \tan (at+P) \quad \ldots\ldots\ldots\ldots\ldots \quad (67)$$

which may be reduced to the form

$$\mathrm{Tan}^3 at = \tan P \quad \ldots\ldots\ldots\ldots\ldots \quad (68)$$

Values of <u>at</u> obtained for different values of <u>P</u> in formula (68) may then be substituted in (65) or (66) to obtain the corresponding critical values of <u>R</u> which determine when the tide will change from semidiurnal to diurnal. These critical values are given in table 3 for each 15° of <u>P</u>. When <u>P</u> equals 0° or 180° and the two low waters are of equal height, both low waters will merge with the LHW to form a single low water, and when <u>P</u> equals 90° or 270° and the two high waters are of equal height they will merge with the HLW to form a single high water.

When the ratio <u>R</u> is less than 2, the compound wave will be semidiurnal for all values of <u>P</u>, and when <u>R</u> is greater than 4, the compound wave will be diurnal for all values of <u>P</u>. When <u>R</u> has a value between 2 and 4, the compound wave may be either diurnal or semidiurnal depending upon the phase relation <u>P</u>.

In general. the predominating constituent in the semidiurnal wave is M_2 while the diurnal wave consists primarily of constituents K_1 and O_1 uniting in different phase relations throughout the tropical month. Although both the semidiurnal and diurnal waves are being continually modified by the presence of other constituents, the effects of the latter are to a large extent averaged out over a long period of time. If we consider only the three principal constituents at this time, the ratio <u>R</u> of the preceding discussion will equal $(K_1+O_1)/M_2$ at the time of the tropic tides and $C(K_1+O_1)/M_2$ at other times during the month; and the phase difference <u>P</u> expressed in degrees of the diurnal wave will equal $\frac{1}{2}(M_2^\circ-K_1^\circ-O_1^\circ)$ at the time of the tropic tides and $\frac{1}{2}(M_2^\circ-K_1^\circ-O_1^\circ)+x$ at other times during the month. For brevity, the ratio $(K_1+O_1)/M_2$ will be written "KO/M", and the phase difference $\frac{1}{2}(M_2^\circ-K_1^\circ-O_1^\circ)$ will be contracted to "MKO".

<u>Acceleration in semidiurnal tide due to diurnal wave.</u>- The times of the high and low waters of the semidiurnal wave may be accelerated or retarded, that is to say, made to occur earlier or later, because of the presence of the diurnal wave. A retardation will be considered as a negative acceleration. There are two high waters and two low waters of the semidiurnal wave which occur within a single period of the diurnal wave

and each of these may be affected differently by the diurnal wave. Therefore, there should be 4 values for time (t) or angle (at) which will satisfy the derived equation (65), provided R and P are within the critical limits for a semidiurnal tide. For convenience of identification, the high water corresponding to the maximum of the semidiurnal wave taken for the time origin will be called the "1st high water" and the succeding tides designated as "1st low water", "2nd high water", and "2nd low water", respectively.

Values of at corresponding to the two maxima of the semidiurnal wave are 0° and 180° and those corresponding to the two minima are 90° and 270°. Letting the accelerations due to the presence of the diurnal wave be a', a", b', and b", respectively, all expressed in degrees of the diurnal wave, the values of at corresponding to the maxima and minima of the compound wave are (0°-a'), (180°-a"), (90°-b'), and (270°-b"). Substituting these values successively in equation (65) and reducing to simple forms, we obtain-

$$2 \sin 2a' = R \sin (P-a') \quad \ldots \quad (69)$$

$$2 \sin 2a'' = -R \sin (P-a'') \quad \ldots \quad (70)$$

$$2 \sin 2b' = -R \cos (P-b') \quad \ldots \quad (71)$$

$$2 \sin 2b'' = R \cos (P-b'') \quad \ldots \quad (72)$$

Equations (69) to (72) are similar in form and a set of numerical values derived from one will be applicable to all with a suitable allowance in the argument P. They do not admit of a simple direct solution, but by a system of trials and approximations, numerical values have been obtained which are included in Tables 8 and 8a, the first containing accelerations for the HHW's and LLW's, and the latter the accelerations for the LHW's and HLW's. In either case the phase difference P is used as the argument when obtaining the high water accelerations but this argument is to be changed by ± 90° for the low water accelerations. Tables 8 and 8a are expressed in degrees of the diurnal wave. Corresponding values reduced to solar hours are given in tables 9 and 9a.

Tropic lunitidal intervals.- The intervals pertaining to the tropic tides may be obtained by applying the accelerations from tables 9 and 9a to the mean intervals from formulas (34) and (35). As arguments for entering these tables, the amplitude ratio may be taken equal to KO/M as a sufficiently close approximation, but for the phase difference the effects of M_4 and M_6 on the semidiurnal wave should be included. These effects are represented by the accelerations v and w in the times of the high and low waters, respectively. Including these accelerations, which are halved for expressing in degrees of the diurnal wave, the phase difference arguments for the tropic tides become MKO-½v for the high water and MKO-½w±90° for the low water. Using the arguments indicated in the tables, the tropic intervals may be expressed by the following formulas-

TcHHWI = Mean HWI - HW acceleration, table 9 . . . . (73)

TcLLWI = Mean LWI - LW acceleration, table 9 . . . . (74)

TcLHWI = Mean HWI - HW acceleration, table 9a. . . . . (75)

TcHLWI = Mean LWI - LW acceleration, table 9a. . . . . (76)

Assuming that the mean HWI in hours is approximately equal to $M_2^\circ/29$, the tropic HHWI will be referred to the "a" transit and the tropic LHWI to the "b" transit when the phase difference MKO-½v is in the 1st or 4th quadrants; otherwise the transit reference will be reversed. If this phase difference is exactly 90° or 270°, the two high waters will be of equal height, one being accelerated and the other retarded. The accelerated time corresponding to the 90° argument or the retarded time corresponding to the 270° argument will be referred to the "a" transit, the other high water in either case being referred to the "b" transit. If the mean HWI differs by approximately 12 hours from $M_2^\circ/29$, the above high water references will be reversed.

Assuming that mean LWI is approximately equal to $(M_2^\circ+180^\circ)/29$, the tropic LLWI will be referred to the "a" transit and the tropic HLWI to the "b" transit when the phase difference MKO-½w is in the 1st or 2nd quadrants; otherwise the transit reference will be reversed. If this phase difference is exactly equal to 0° or 180°, the two low waters will be of equal height, one being retarded and the other accelerated. The retarded time corresponding to the 0° difference or the accelerated time corresponding to the 180° difference will be referred to the "a" transit, the other low water in either case being referred to the "b" transit. If the mean LWI differs by approximately 12 hours from $(M_2^\circ+180^\circ)/29$, all of the above low water references will be reversed.

The "b" intervals can be referred to the "a" transit by the addition or subtraction of 12.42 hours, and for the purpose of uniformity it is recommended that all tropic intervals be referred to the "a" transit.

The average of the Lunitidal intervals pertaining to the higher high waters or to the lower low waters over a period of a month or more is known as the mean higher high water interval or mean lower low water interval. Such averages, however usually include intervals referred to both "a" and "b" transits, the mean being marked according to the reference at the time of the tropic tides. The incongruity of including in the means intervals referred to different transits serves to obscure any definition of the quantities and therefore no attempt is made here to derive them from the harmonic constants.

<u>Maximum and minimum heights of compound wave.</u>- By substituting in formula (64) the values of <u>at</u> corresponding to the times of the maxima and

minima, the corresponding heights of the high and low waters may be obtained, these heights being referred to the mean water level. Let H' and H'' represent the heights of the 1st and 2nd high waters, and L' and L'' the heights of the 1st and 2nd low waters. The values of at corresponding to the times of these heights are $(0°-a')$, $(180°-a'')$, $(90°-b')$, and $(270°-b'')$, respectively. Making the substitutions in formula (64) and reducing, we obtain-

$$H' = \cos 2a' + R \cos (P-a') \quad \ldots \quad (77)$$

$$H'' = \cos 2a'' - R \cos (P-a'') \quad \ldots \quad (78)$$

$$L' = -\cos 2b' - R \sin (P-b') \quad \ldots \quad (79)$$

$$L'' = -\cos 2b'' + R \sin (P-b'') \quad \ldots \quad (80)$$

In the above formulas the amplitude of the semidiurnal wave is taken as unity and the values obtained are to be used as factors to be applied to the actual amplitude of the semidiurnal wave, the average magnitude of the latter being approximately the amplitude of constituent M_2. Factors for the higher high waters are given by formula (77) when <u>P</u> is in the 1st or 4th quadrant and by formula (78) when <u>P</u> is in the 2nd or 3rd quadrant, while factors for the lower high waters are given by the same formulas when <u>P</u> is in one of the opposite quadrants. Factors for the lower low waters are given by formula (79) when <u>P</u> is in the 1st or 2nd quadrant and by formula (80) when <u>P</u> is in the 3rd or 4th quadrant. Factors for the higher low waters are given by the same formulas when <u>P</u> is in one of the opposite quadrants. The high water factors, however, are applicable to the low waters if the phase difference is changed by 90° and the sign of the factor reversed. Factors in table 10 are applicable to the higher high and lower low waters, and factors in table 10a are applicable to the lower high and higher low waters.

<u>Sequence of tide.</u>- The sequence in which the HHW, LHW, HLW, and LLW occur will depend upon the phase difference <u>P</u> in the above formulas. The order of occurrence is shown in the following table. In certain cases the two high waters or the two low waters will be of equal height and these are indicated by the asterisk (*) in the table. There are also shown which tides are accelerated and which ones are retarded, the accelerated tides being marked by the plus (+) sign and the retarded tides by the minus (−) sign, while a (o) indicates no change in time due to the diurnal wave.

P	H'	L'	H"	L"
0°	HHW o	LW* −	LHW o	LW* +
1st quadrant	HHW +	LLW −	LHW −	HLW +
90°	HW* +	LLW o	HW* −	HLW o
2nd quadrant	LHW +	LLW +	HHW −	HLW −
180°	LHW o	LW* +	HHW o	LW* −
3rd quadrant	LHW −	HLW +	HHW +	LLW −
270°	HW* −	HLW o	HW* +	LLW o
4th quadrant	HHW −	HLW −	LHW +	LLW +

Height inequalities.- The tropic high water inequality (HWQ) is the difference in the height of the two high waters of the day at the time of the tropic tides, and the tropic low water inequality (LWQ) is the corresponding difference in height of the two low waters. The mean diurnal high water inequality (DHQ) is the difference in height between the mean of all higher high waters and the mean of all high waters over a period of a month or more and is therefore one-half the difference between the mean of the higher high waters and lower high waters. Similarly, the mean diurnal low water inequality (DLQ) is the difference between the mean of the lower low waters and of all low waters, or one-half the difference between the mean lower low and the mean higher low water. Table 11 contains diurnal inequality factors derived by differences from tables 10 and 10a. The tabular factor multiplied by the amplitude of the semidiurnal wave will give the difference between the higher high and lower high or between the higher low and lower low water. For the low water inequality, the phase difference is taken 90° different from that used for the high water inequality. With certain phase relations, the height inequalities may be modified by the presence of constituent P_1, the effects of which are not included in the factors of table 11 but will be given further consideration.

Relation mean tide level to mean water level.- Although for practical purposes the mean tide level obtained by averaging the heights of the high and low waters is often taken as the approximate equivalent of mean water level derived from the hourly heights of the tide, the two datums may at times differ as much as a tenth of a foot or more. The two principal causes contributing to this difference are the harmonic M_4 and the diurnal wave represented by constituents K_1 and O_1.

For the effect of M_4, let P_4 = phase relation $(2M_2^\circ - M_4^\circ)$, and v' and w' the accelerations in the semidiurnal high and low waters, respectively, due to the presence of M_4. Considering only the principal semidiurnal constituent M_2 and its harmonic M_4, the resultant high and low waters referred to the mean water level may be expressed as follows-

$$HW = M_2 \cos v' + M_4 \cos (P_4 - 2v') \quad \ldots\ldots \quad (81)$$

$$LW = -M_2 \cos w' + M_4 \cos (P_4 - 2w') \quad \ldots\ldots \quad (82)$$

Then for the elevation of mean tide level-

$$\tfrac{1}{2}(HW+LW) = \tfrac{1}{2}M_2(\cos v' - \cos w')$$

$$+ M_4 \cos (v'-w') \cos (P_4 - v' - w') \quad \ldots\ldots \quad (83)$$

As v' and w' are usually small with opposite signs, the above may be represented approximately by the simplified expression-

$$\tfrac{1}{2}(HW+LW) = M_4 \cos P_4 = M_4 \cos (2M_2^\circ - M_4^\circ) \quad \ldots\ldots \quad (84)$$

It will be noted that (84) is positive with an upward displacement of mean tide level when ($2M_2^\circ - M_4^\circ$) is in the 1st or 4th quadrants, and negative with a downward displacement when this phase difference is in the 2nd or 3rd quadrants.

For the effect of the diurnal wave, let R and P represent respectively the amplitude ratio and the high water phase difference between the diurnal and semidiurnal waves. The elevation of mean tide level above mean water level due to the presence of the diurnal wave may then be expressed by the following formula-

$$\tfrac{1}{4}(HHW+LHW+HLW+LLW) = -1/16\ M_2\ R^2 \cos 2P \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (85)$$

For the tropic tides, R may be taken as approximately equal to KO/M and P equal to MKO. At other times during the tropical month, the ratio R will be modified by the factor C from table 6 and the phase relation P will be modified by the difference x from table 7, these modifications depending upon the ratio O_1/K_1. If this ratio is taken as 0.7 in accordance with the theoretical coefficients of the constituents, it will be found that the mean value of formula (85) for the entire month is 0.485 times its value at the time of the tropic tides. Making the necessary substitutions, we then have for the mean displacement of the mean tide level due to the diurnal wave the following-

$$\tfrac{1}{4}(HHW+LHW+HLW+LLW) = -0.030\ M_2\ (KO/M)^2 \cos 2(MKO)$$

$$= -0.030\ (K_1+O_1)(KO/M) \cos 2(MKO) \quad . \ . \ . (86)$$

Although the numerical coefficient in the above formula is based upon an assumed ratio of 0.7 for O_1/K_1, it may be used without material error when O_1/K_1 or its reciprocal has any value between 0.5 and 1.0. The following schedule gives the coefficients corresponding to different ratios for O_1/K_1 or its reciprocal, the ratio being followed by the coefficient in parentheses; 0.0 (0.000), 0.1 (.009), 0.2 (.017), 0.3 (.022), 0.4 (.026), 0.5 (.028), 0.6 (.029), 0.7 (.030), 0.8 (.031), 0.9 (.031), 1.0 (.031).

Combining (84) and (86) into a single formula to include the effects of both disturbing influences, we have-

$$MTL-MWL = M_4 \cos (2M_2^\circ - M_4^\circ) - 0.03\ (K_1+O_1)(KO/M) \cos 2(MKO) \quad . \ . (87)$$

Effect of P_1.- The principal solar diurnal constituent P_1 has a speed that is incommensurable with that of the lunar diurnal wave and for this reason its effects on the latter may be expected to be averaged out to considerable extent in a series of observations extending over a year or more. However, since in the tabulations of the observation the HHW's, LHW's, HLW's and LLW's are selected according to their actual relative heights rather than by the theoretical sequence of occurrence, the mean height of any group may be affected when the amplitude of P_1 is sufficiently large to reverse the sequence.

The following is based upon a discussion in Harris's Manual of Tides, Part III, pages 145-146. For convenience, reference is made to the high waters and to the high water inequality but the discussion applies equally well to the low waters and the low water inequality.

Let Q' = difference HHW-LHW due to lunar diurnal wave (table 11),

Q = corresponding difference including P_1 effect

$L = Q'/2P_1$. Then $Q' = 2P_1L$ . . . . (88)

When $\underline{L}$ is greater than unity, $\underline{Q'}$ then being greater than $2P_1$, the latter cannot cause a reversal in the order of the two high waters and their mean heights and inequality will be unaffected. When $\underline{L}$ is less than unity, a reversal may or may not take place depending upon the phase relation between the solar and lunar waves at the time, this phase relation changing continually throughout the tropical year.

Let $\underline{X}$ = phase of P_1 at the time of one of the semidiurnal high waters. The phase of P_1 at the following semidiurnal high water will then be approximately $(X+180°)$. One of these high waters will then be increased in height by $P_1 \cos X$ and the other lowered by the same amount, the diurnal inequality that P_1 tends to introduce being $2P_1 \cos X$. When $\underline{X} = \cos^{-1}L$, this inequality equals $2P_1L$ or Q'. The P_1 inequality, always taken as positive, will therefore exceed Q' when $\underline{X}$ falls between the limits 0° and $\pm \cos^{-1}L$ or between the limits 180° and $180° \pm \cos^{-1}L$, and will be less than Q' during the remainder of the cycle.

The average value of $\cos X$ between the limits $X=0$ and $X=\cos^{-1}L$ is $(1-L^2)^{1/2}/\cos^{-1}L$ with $\cos^{-1}L$ expressed in the radian unit. The average P_1 inequality during the period that it exceeds Q' is therefore $2P_1(1-L^2)^{1/2}/\cos^{-1}L$, and during this period in which the P_1 inequality predominates over the lunar inequality, the latter may be considered as being approximately averaged out. For the remainder of the cycle, the Q' inequality predominates with the effect of P_1 averaged out. The respective weights to be given to the two parts to obtain a mean for the entire cycle are $2 \cos^{-1}L$ and $(\pi - 2 \cos^{-1}L)$. The mean inequality corrected for P_1 may then be written-

$$Q = 1/\pi[4P_1(1-L^2)^{1/2} + Q' \; (\pi - 2 \cos^{-1}L)] \text{. . . . . (89)}$$

in which $\cos^{-1}L$ is expressed in the radian unit.

Transposing, combining with (88), and expressing $\cos^{-1}L$ in degrees, the following may be obtained-

$$Q = Q' + 2P_1 \; [(2/\pi)(1-L^2)^{1/2} - (1/90) \; L \; (\cos^{-1}L)°]$$

$$= Q' + 2P_1 \; [0.6366(1-L^2)^{1/2} - 0.0111 \; L \; (\cos^{-1}L)°]$$

$$= Q' + 2P_1 \times \text{table 12} \text{ . . . . (90)}$$

in which

$$\text{Table } 12 = 0.6366\ (1-L^2)^{1/2} - 0.0111\ L(\cos^{-1}L)^\circ \quad \ldots\ldots \quad (91)$$

Thus, the inequality is increased by an amount equal to the product of $2P_1$ and the table 12 factor, the height of the HHW being increased and that of the LHW diminished by P_1 x table 12.

The formulas and tables are also applicable to the low water inequality by letting Q' and Q represent the corresponding differences in the low water heights. In this case the height of the LLW will be diminished and that of the HLW increased by the product of P_1 and table 12.

It will be noted that since the amplitude of the lunar diurnal wave varies throughout the tropical month, the relative effect of P_1 will also vary and be greatest at the time of the equatorial tides when the amplitude of the lunar wave is a minimum. Based upon the theoretical coefficients, the amplitude of P_1 is 0.193 times the amplitude (K_1+O_1) for the tropic tides and 1.145 times (K_1-O_1) for the equatorial tides.

Tropic heights and inequalities.- By using the theoretical relation of P_1 to the amplitude of the lunar diurnal wave at the time of the tropic tides, the inequality factors in table 11 may be modified by means of formula (90) to include the P_1 effect. For the tropic tides the ratio argument in table 11 is approximately KO/M, with the amplitude of M_2 taken as unity. The corresponding value of $2P_1$ for each line of the table is then obtained by multiplying the argument by the factor 0.386. Comparing these values of $2P_1$ with the tabular values, which represent the quantity Q' of formula (88), it will be found that only in the last two columns the tabular values are less than $2P_1$ and therefore these will be the only columns affected. The factors in these columns have been corrected by means of formula (90) and the corrected values incorporated in table 11t, the latter table being designed for the computation of the tropic inequalities. In using this table the ratio argument may be taken as KO/M, and the phase difference as MKO–½v for the high water inequality and MKO–½w$\pm$90° for the low water inequality, the factor itself then being multiplied by the amplitude of M_2. These inequalities may then be expressed by the following formulas-

$$HWQ = M_2 \times (\text{HW factor, table 11-t}) \quad \ldots\ldots \quad (92)$$

$$LWQ = M_2 \times (\text{LW factor, table 11-t}) \quad \ldots\ldots \quad (93)$$

Table 10 for the heights of the HHW and LLW may also be modified for the effects of P_1 at the time of tropic tides by adding to the tabular value one-half the correction used in modifying table 11, the last two columns only being affected. All columns of table 10 are carried to an amplitude ratio of 4.0 although the tide may become diurnal with certain phase relations after the ratio exceeds 2.0. The P_1 correction is based upon an inequality relation which vanishes when the tide becomes diurnal.

However, for the phase relation of 90° and its multiples, the tide continues to be semidiurnal up to the amplitude ratio of 4.0 and computations of the P_1 corrections for the last column of the table were made accordingly. For the phase relation of 80°, the tide becomes diurnal at approximately the amplitude ratio of 2.7. Above this ratio the P_1 effects were inferred from other considerations. The modified factors have been incorporated in table 10t which is designed for the computation of the height of the tropic HHW above mean water level or the depression of the tropic LLW below this datum, the same arguments being used as for table 11t. If the heights of LHW and HLW are desired, they may be obtained by applying the tropic inequalities to the heights of the HHW and LLW. Formulas for the tropic heights as referred to the mean water level are as follows-

$$TcHHW = M_2 \times (\text{HW factor, table 10t}) \ldots\ldots\ldots\ldots (94)$$

$$TcLLW = -M_2 \times (\text{LW factor, table 10t}) \ldots\ldots\ldots\ldots (95)$$

$$TcLHW = TcHHW - HWQ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

$$TcHLW = TcLLW + LWQ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (97)$$

Mean heights and inequalities.- Table 13 contains factors which are to be multiplied by (K_1+O_1) to obtain the mean diurnal inequalities. The factors include the effect of P_1. The amplitude and phase relation between the diurnal and semidiurnal wave continually change throughout the tropical month and the factors in table 13 were derived from the basic factors in table 11 with arguments modified for the different times of the month by means of tables 6 and 7. Corrections for P_1 were obtained from table 12. The factors were first computed for an assumed ratio of unity for KO/M, and by checking were found to apply approximately for other values of this ratio. It is believed that the factors can be used without important error up to a ratio of 4.0 for KO/M. Although the tropic tides may be diurnal when KO/M has a value between 2.0 and 4.0, the tide during a large portion of the remainder of the month may be expected to be semidiurnal. A mean diurnal inequality covering a period when the tide is partly diurnal and partly semidiurnal, does not have the same significance as one including only semidiurnal tides but may be used as a step in the computation of the HHW and LLW heights.

If the amplitude of O_1 is greater than K_1, the ratio K_1/O_1 instead of O_1/K_1 may be used as the argument in entering table 13. The phase relation P is to be taken as MKO–½v for the high water inequality and as MKO–½w±90° for the low water inequality. The mean inequalities may then be expressed by the following formulas-

$$DHQ = (K_1+O_1) \times (\text{HW factor, table 13}) \ldots\ldots\ldots\ldots (98)$$

$$DLQ = (K_1+O_1) \times (\text{LW factor, table 13}) \ldots\ldots\ldots\ldots (99)$$

The mean heights of HHW, LLW, LHW, and HLW may be obtained by combining the mean inequalities with the half range of tide and including the difference (MTL–MWL) for reference to mean water level. These heights may then be expressed by the following formulas-

$$\text{Mean HHW} = \tfrac{1}{2}\,Mn + DHQ + (MTL-MWL) \quad (100)$$

$$\text{Mean LLW} = -\tfrac{1}{2}\,Mn - DLQ + (MTL-MWL) \quad (101)$$

$$\text{Mean LHW} = \tfrac{1}{2}\,Mn - DHQ + (MTL-MWL) \quad (102)$$

$$\text{Mean HLW} = -\tfrac{1}{2}\,Mn + DLQ + (MTL-MWL) \quad (103)$$

Diurnal tide.- When the ratio KO/M exceeds 4, the tide may be considered as predominatingly diurnal and will be treated as such. Taking the diurnal wave amplitude as unity, let-

R' = (semidiurnal wave amplitude)/(diurnal wave amplitude)

d = acceleration in diurnal HW due to semidiurnal wave

d' = acceleration in diurnal LW due to semidiurnal wave

Both $\underline{d}$ and $\underline{d'}$ are expressed in degrees of the diurnal wave. Let other symbols represent the same quantities as previously, $\underline{P}$ being the phase of the diurnal wave corresponding to one of the high waters of the semidiurnal wave. To avoid an ambiguity arising from the fact that there are two semidiurnal high waters within the period of the diurnal wave, the difference $\underline{P}$ will be referred to the first high water following the "a" transit of the mean moon. With this reference, $\underline{P}$ at the time of the tropic tides will equal MKO. Taking the high water of the diurnal wave as the time origin, the equation of the sum of the two waves may be written-

$$y = \cos at + R' \cos (2at-2P) \quad (104)$$

and its first derivative equated to zero is as follows-

$$\sin at + 2R' \sin (2at-2P) = 0 \quad (105)$$

Substituting successively in (105) values of $\underline{at}$ corresponding to the high and low water of the compound wave, these being $(0°-d)$ and $(180°-d')$, respectively, we have

$$\sin d = -2R' \sin (2P+2d) \quad (106)$$

$$\sin d' = 2R' \sin (2P+2d') \quad (107)$$

Values for acceleration d from equation (106) are compiled in table 14 and corresponding values expressed in solar hours are given in table 15, the arguments for entering these tables being R' and the phase difference P. These tables may be used also for the low water acceleration d' by taking the phase difference argument as P±90°.

The high and low water heights of the compound wave may be expressed by the following formulas, the amplitude of the diurnal wave being unity and the heights being referred to the mean water level-

$$HW = \cos d + R' \cos (2P+2d) \quad \ldots \quad (108)$$

$$LW = -\cos d' + R' \cos (2P+2d') \quad \ldots \quad (109)$$

High water factors from formula (108) are compiled in table 16. The same table may be used for the low water factors of formula (109) by taking P±90° as the phase difference argument and applying the negative sign to the tabular value.

Tropic diurnal intervals.- The tropic high water interval of the diurnal wave expressed in hours and referred to the "a" transit equals $\frac{1}{2}(K_1^\circ+O_1^\circ)/14.492$, or $0.0345(K_1^\circ+O_1^\circ)$ and the low water interval referred to the same transit equals $0.0345(K_1^\circ+O_1^\circ) \pm 12.42$ hours. In the above expressions, K_1° and O_1° must differ by less than 180°, the smaller one being increased by 360° if necessary. To these intervals there are to be applied the accelerations due to the semidiurnal wave which are given in table 15, using as arguments $R' = M_2/(K_1+O_1)$ and P = MKO for high water and MKO±90° for low water acceleration. The corrected intervals expressed in hours are given by the following formulas-

$$\text{Tc diurnal HWI} = 0.0345(K_1^\circ+O_1^\circ)-(\text{HW accel. table 15}) \quad \ldots \quad (110)$$

$$\text{Tc diurnal LWI} = 0.0345(K_1^\circ+O_1^\circ)-(\text{LW accel. table 15}) \pm 12.42 \quad .(111)$$

From an examination of table 15 it will be noted that the high and low water accelerations for any value of P will have opposite signs. When P is in the 1st or 3rd quadrants, the high waters will be retarded and the low waters accelerated thus lengthening the duration of rise, and when P is in the 2nd or 4th quadrants, the high waters are accelerated and the low waters retarded thus lengthening the duration of fall.

Tropic diurnal heights.- Table 16 contains factors which, when multiplied by the amplitude of the diurnal wave, will give the heights of high and low waters referred to the mean water level. For the tropic tides the amplitude of the diurnal wave may be taken as (K_1+O_1), argument R' equal to $M_2/(K_1+O_1)$, and phase difference P equal to MKO for the HW factor and MKO±90° for the LW factor. The tropic heights are then expressed by the following formulas-

$$\text{Tc diurnal HW} = (K_1+O_1) \times (\text{HW factor, table 16}) \quad \ldots \quad (112)$$

$$\text{Tc diurnal LW} = -(K_1+O_1) \times (\text{LW factor, table 16}) \quad \ldots \quad (113)$$

Mean diurnal heights.- Table 17 contains factors which, when multiplied by (K_1+O_1), are designed to give the mean diurnal high and low water heights as referred to mean water level. The table is applicable when the tide is predominatingly diurnal, and during portions of the month when the tide may become semidiurnal only the HHW's and LLW's are taken into account. Table 17 has been prepared from the basic factors contained in tables 10 and 16, using arguments modified for different times of the month by means of tables 6 and 7. In the preparation of the table, the ratio O_1/K_1 was taken at its theoretical value of 0.7 but the table is applicable without material error for other values of this ratio. The effect of P_1 will vary throughout the month and there has been incorporated in table 17 an empirical adjustment, based upon comparisons with observed data, to take account of the effects of P_1 and other unknown elements. The arguments to be used in entering table 17 are $R' = M_2/(K_1+O_1)$ and $P = MKO$ for HW factor and $MKO \pm 90°$ for LW factor. Formulas for the mean diurnal heights follow.

$$\text{Mean diurnal HW} = (K_1+O_1) \times (\text{HW factor, table 17}) \quad . \; . \; . \; . \quad (114)$$

$$\text{Mean diurnal LW} = -(K_1+O_1) \times (\text{LW factor, table 17}) \quad . \; . \; . \; . \quad (115)$$

Form 180.- This form based on the preceding discussion has been devised to facilitate the computation of certain non-harmonic tidal constants. It is designed to take care of three classes of tides; semidaily tides with relatively small diurnal inequality, semidaily tides with large diurnal inequality and daily tides for which the ratio of K_1+O_1 to M_2 is 4 or more. An example of each class is given on the following three pages.

Form 180
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
(Rev. Nov. 1948)

TIDES: HARMONIC CONSTANT REDUCTIONS

Station Bristol, Rhode Island *Lat.* 41° 40' N. *Long.* 71° 16' W.

Length of series 1 year days. Series begins 1890, Aug. 6 Source of constants U.S.C.& G.S. Analysis

Epochs and intervals referred to ~~Greenwich~~ Local meridian. Heights referred to mean water level.

Harmonic constants

	H		κ
K_1	0.21	K_1°	94
O_1	0.16	O_1°	131
P_1	0.09	P_1°	94
μ_2		μ_2°	
S_2	0.44	S_2°	245
M_2	1.90	M_2°	223
M_4	0.29	M_4°	135
M_6	0.04	M_6°	245
N_2	0.42	N_2°	206

Amplitude relations

(1)	$M_4 + M_2$	0.15
(2)	$M_6 + M_2$	0.02
(3)	$S_2 + M_2$	0.23
(4)	$N_2 + M_2$	0.22
(5)	$O_1 + K_1$	0.76
(6)	$K_1 + O_1$	0.37
(7)	$(K_1 + O_1) + M_2$	0.19

Epoch relations

If the direct difference between the constituents of (10) to (13) is more than 180°, add 360° to the lesser one before subtracting or adding. Use the negative sign, if necessary.

Rejection of 720° (but not 360°) admissible in (13) and (14).

(8)	$2M_2^\circ - M_4^\circ$	311
(9)	$3M_2^\circ - M_6^\circ$	64
(10)	$S_2^\circ - M_2^\circ$	22
(11)	$M_2^\circ - N_2^\circ$	17
(12)	$K_1^\circ - O_1^\circ$	−37
(13)	$K_1^\circ + O_1^\circ$	225
(14)	$M_2^\circ - K_1^\circ - O_1^\circ - M_2^\circ - (13)$	−2
(15)	MKO = (14) ÷ 2	−1

Age of inequalities in hours

(16)	Phase age = 0.984 × (10)	22
(17)	Parallax age = 1.837 × (11)	31
(18)	Diurnal age = 0.911 × (12)	−34

Mean intervals

(19)	v' = Table 1, args. (1) and (8)	−8.9
(20)	w' = Table 1, args. (1), (8) ± 180°	+17.3
(21)	(9) − 3(19)	91
(22)	(9) − 3(20)	12
(23)	v" = Table 2, args. (2), (21)	3.4
(24)	w" = Table 2, args. (2), (22)	0.6
(25)	v = (19) + (23)	−6
(26)	w = (20) + (24)	+18

(27)	$M_2^\circ - (25)$	229
(28)	$M_2^\circ - (26) \pm 180°$	25
(29)	HWI = 0.0345 × (27)	7.90
(30)	LWI = 0.0345 × (28)	0.86

Mean range

(31)	Cos v = cos (25)	0.995
(32)	Cos w = cos (26)	0.951
(33)	Table 4, arg. (3)	0.051
(34)	Table 5, arg. (7)	0.003
(35)	(31) + (32) + (33) + (34)	2.000
(36)	Cos [(8) − 2v]	+0.799
(37)	Cos [(8) − 2w]	+0.087
(38)	Cos [(9) − 3v]	+0.139
(39)	Cos [(9) − 3w]	+0.985
(40)	$1.02M_2 \times (35)$	3.88
(41)	$1.02M_4[(36) - (37)]$	0.21
(42)	$1.02M_6[(38) + (39)]$	0.05
(43)	Mn = (40) + (41) + (42)	4.14

Spring and neap range

(44)	$0.536\,S_2 \times (3)$	0.05
(45)	$S_2 + \mu_2 \cos(2M_2^\circ - S_2^\circ - \mu_2^\circ)$	0.44
(46)	$1.96 - 0.06 \times (7)^2$	1.96
(47)	(45) × (46)	0.86
(48)	Sg = (43) − (44) + (47)	4.95
(49)	Np = (43) − (44) − (47)	3.23
(50)	Sg + Mn = (48) ÷ (43)	1.20
(51)	Np + Mn = (49) ÷ (43)	0.78

Perigean and apogean range

(52)	Pn + Mn = 1 + (4)	1.22
(53)	An + Mn = 1 − 0.75 × (4)	0.84
(54)	Pn = (43) × (52)	5.05
(55)	An = (43) × (53)	3.48

Tropic intervals

(56)	(15) − ½(25)	
(57)	(15) − ½(26) − 90°	
(58)	Table 9, args. (7), (56)	
(59)	Table 9, args. (7), (57)	
(60)	Table 9a, args. (7), (56)	
(61)	Table 9a, args. (7), (57)	
(62)	TcHHWI = (29) − (58)	
(63)	TcLLWI = (30) − (59)	
(64)	TcLHWI = (29) − (60)	
(65)	TcHLWI = (30) − (61)	

HHWI refers to transit *a* when (56) is in 1st or 4th quadrant, if this interval is approximately M_2° + 29.
LLWI refers to transit *a* when (57) is in 1st or 4th quadrant, if this interval is approximately (M_2° + 180°) + 29.

Tropic heights

(66)	Table 11, args. (7), (56)	
(67)	Table 11, args. (7), (57)	
(68)	Table 10, args. (7), (56)	
(69)	Table 10, args. (7), (57)	
(70)	HWQ = M_2 × (66)	
(71)	LWQ = M_2 × (67)	
(72)	TcHHW = M_2 × (68)	
(73)	TcLLW = − M_2 × (69)	
(74)	TcLHW = (72) − (70)	
(75)	TcHLW = (73) + (71)	
(76)	Gc = (72) − (73)	

Mean tide level

(77)	M_4 cos (6)	0.19
(78)	0.03 × (6) × (7) × cos (14)	
(79)	MTL = (77) − (78)	+0.19

Mean heights

(80)	Table 13, args. (5), (56)	
(81)	Table 13, args. (5), (57)	
(82)	DHQ = (6) × (80)	
(83)	DLQ = (6) × (81)	
(84)	MHHW = ½(43) + (79) + (82)	
(85)	MLLW = − ½(43) + (79) − (83)	
(86)	MLHW = (84) − 2(82)	
(87)	MHLW = (85) + 2(83)	
(88)	Gt = (84) − (85)	

Diurnal tide
when (7) is greater than 4

(89)	R' = reciprocal of (7)	
(90)	MKO ± 90° = (15) ± 90°	
(91)	Table 15, args. (89), (15)	
(92)	Table 15, args. (89), (90)	
(93)	0.0345 (13)	
(94)	TcHHWI = (93)	
(95)	TcLLWI = (93) − (92) ± 12.42	
(96)	Table 16, args. (89), (15)	
(97)	Table 16, args. (89), (90)	
(98)	Table 17, args. (89), (15)	
(99)	Table 17, args. (89), (90)	
(100)	TcHHW = (6) × (96)	
(101)	TcLLW = − (6) × (97)	
(102)	MHHW = (6) × (98)	
(103)	MLLW = − (6) × (99)	
(104)	Gc = (100) − (101)	
(105)	Gt = (102) − (103)	

Computed by BWW Date 5/25/50 Verified by RAC Date 5/25/50

U. S. GOVERNMENT PRINTING OFFICE 818498

Form 169
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
(Rev. Nov. 1948)

TIDES: HARMONIC CONSTANT REDUCTIONS

Station ......Avila, California... *Lat.* 35° 10!2 N.. *Long.* 120° 44!4 W.

Length of series .....369........ days. Series begins 1949..1-1-0...... Source of constants ....U. S. C. & G. S. Anaylsis

Epochs and intervals referred to ~~Greenwich~~ Local meridian. Heights referred to mean water level.

	Harmonic constants		
	H		Gr. κ
K_1	1.16	K_1°	94
O_1	0.73	O_1°	78
P_1	0.38	P_1°	90
μ_2	0.04	μ_2°	217
S_2	0.48	S_2°	281
M_2	1.60	M_2°	287
M_4	0.01	M_4°	100
M_6	0.01	M_6°	286
N_2	0.35	N_2°	261

	Amplitude relations	
(1)	$M_4 \div M_2$	0.01
(2)	$M_6 \div M_2$	0.01
(3)	$S_2 \div M_2$	0.30
(4)	$N_2 \div M_2$	0.22
(5)	$O_1 \div K_1$	0.63
(6)	$K_1 + O_1$	1.89
(7)	$(K_1+O_1) \div M_2$	1.18

Epoch relations

If the direct difference between the constituents of (10) to (13) is more than 180°, add 360° to the lesser one before subtracting or adding. Use the negative sign, if necessary.

Rejection of 720° (but not 360°) admissible in (13) and (14).

(8)	$2M_2^\circ - M_4^\circ$	114
(9)	$3M_2^\circ - M_6^\circ$	215
(10)	$S_2^\circ - M_2^\circ$	−6
(11)	$M_2^\circ - N_2^\circ$	26
(12)	$K_1^\circ - O_1^\circ$	16
(13)	$K_1^\circ + O_1^\circ$	172
(14)	$M_2^\circ - K_1^\circ - O_1^\circ = M_2^\circ - (13)$	115
(15)	MKO = (14) ÷ 2	58
	Age of inequalities in hours	
(16)	Phase age = 0.984 × (10)	−6
(17)	Parallax age = 1.837 × (11)	48
(18)	Diurnal age = 0.911 × (12)	15
	Mean intervals	
(19)	v′ = Table 1, args. (1) and (8)	1.1
(20)	w′ = Table 1, args. (1), (8) ± 180°	−1.1
(21)	(9) − 3(19)	212
(22)	(9) − 3(20)	218
(23)	v″ = Table 2, args. (2), (21)	−1.0
(24)	w″ = Table 2, args. (2), (22)	−1.1
(25)	v = (19) + (23)	0
(26)	w = (20) + (24)	−2

(27)	M_2° − (25)	287
(28)	M_2° − (26) ± 180°	109
(29)	HWI = 0.0345 × (27)	9.90
(30)	LWI = 0.0345 × (28)	3.76
	Mean range	
(31)	Cos v = cos (25)	1.000
(32)	Cos w = cos (26)	0.999
(33)	Table 4, arg. (3)	0.072
(34)	Table 5, arg. (7)	0.100
(35)	(31) + (32) + (33) + (34)	2.171
(36)	Cos [(8) − 2v]	−0.407
(37)	Cos [(8) − 2w]	−0.469
(38)	Cos [(9) − 3v]	−0.819
(39)	Cos [(9) − 3w]	−0.755
(40)	$1.02M_2$ × (35)	3.54
(41)	$1.02M_4$[(36) − (37)]	0.00
(42)	$1.02M_6$[(38) + (39)]	−0.02
(43)	Mn = (40) + (41) + (42)	3.52
	Spring and neap range	
(44)	0.536 S_2 × (3)	0.08
(45)	$S_2 + \mu_2$ cos ($2M_2^\circ - S_2^\circ - \mu_2^\circ$)	0.49
(46)	1.96 − 0.08 × (7)²	1.85
(47)	(45) × (46)	0.91
(48)	Sg = (43) − (44) + (47)	4.35
(49)	Np = (43) − (44) − (47)	2.53
(50)	Sg ÷ Mn = (48) ÷ (43)	1.24
(51)	Np ÷ Mn = (49) ÷ (43)	0.72
	Perigean and apogean range	
(52)	Pn ÷ Mn = 1 + (4)	1.22
(53)	An ÷ Mn = 1 − 0.75 × (4)	0.84
(54)	Pn = (43) × (52)	4.29
(55)	An = (43) × (53)	2.96
	Tropic intervals	
(56)	(15) − ½(25)	58
(57)	(15) − ½(26) − 90°	329
(58)	Table 9, args. (7), (56)	0.86
(59)	Table 9, args. (7), (57)	−0.47
(60)	Table 9a, args. (7), (56)	−1.19
(61)	Table 9a, args. (7), (57)	0.81
(62)	TcHHWI = (29) − (58) (a)	9.04
(63)	TcLLWI = (30) − (59) (b)	4.23
(64)	TcLHWI = (29) − (60)	11.09
(65)	TcHLWI = (30) − (61)	2.95

HHWI refers to transit a when (56) is in 1st or 4th quadrant, if this interval is approximately M_2° + 29.
LLWI refers to transit a when (57) is in 1st or 4th quadrant, if this interval is approximately (M_2° + 180°) + 29.

	Tropic heights	
(66)	Table 11t, args. (7), (56)	1.207
(67)	Table 11t, args. (7), (57)	1.996
(68)	Table 10t, args. (7), (56)	1.732
(69)	Table 10t, args. (7), (57)	2.048
(70)	HWQ = M_2 × (66)	1.93
(71)	LWQ = M_2 × (67)	3.19
(72)	TcHHW = M_2 × (68)	2.77
(73)	TcLLW = −M_2 × (69)	−3.28
(74)	TcLHW = (72) − (70)	0.84
(75)	TcHLW = (73) + (71)	−0.09
(76)	Gc = (72) − (73)	6.05
	Mean tide level	
(77)	M_4 cos (8)	0.00
(78)	0.03 × (6) × (7) × cos (14)	−0.03
(79)	MTL = (77) − (78)	0.03
	Mean heights	
(80)	Table 13, args. (6), (56)	0.366
(81)	Table 13, args. (6), (57)	0.553
(82)	DHQ = (6) × (80)	0.69
(83)	DLQ = (6) × (81)	1.05
(84)	MHHW = ½(43) + (79) + (82)	2.48
(85)	MLLW = −½(43) + (79) − (83)	−2.78
(86)	MLHW = (84) − 2(82)	1.10
(87)	MHLW = (85) + 2(83)	−0.68
(88)	Gt = (84) − (85)	5.26
	Diurnal tide when (7) is greater than 4	
(89)	R′ = reciprocal of (7)	
(90)	MKO ± 90° = (15) ± 90°	
(91)	Table 15, args. (89), (15)	
(92)	Table 15, args. (89), (90)	
(93)	0.0345 (13)	
(94)	TcHHWI = (93) − (91)	
(95)	TcLLWI = (93) − (92) ± 12.42	
(96)	Table 16, args. (89), (15)	
(97)	Table 16, args. (89), (90)	
(98)	Table 17, args. (89), (15)	
(99)	Table 17, args. (89), (90)	
(100)	TcHHW = (6) × (96)	
(101)	TcLLW = −(6) × (97)	
(102)	MHHW = (6) × (98)	
(103)	MLLW = −(6) × (99)	
(104)	Gc = (100) − (101)	
(105)	Gt = (102) − (103)	

Computed by ...... ECP ...... Date 6/26/51..... Verified by .... MAW .... Date 6/21/51.....

U. S. GOVERNMENT PRINTING OFFICE 815466

Form 189
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
(Rev. Nov. 1948)

TIDES: HARMONIC CONSTANT REDUCTIONS

Station Pensacola, Florida *Lat.* 30° 24!2 N. *Long.* 87° 12!8 W.

Length of series 369 days. Series begins 1939-1-1-0 Source of constants U.S.C.&G.S. Analysis

Epochs and intervals referred to ~~Greenwich~~ Local meridian. Heights referred to mean water level.

Harmonic constants

	H		~~Gre~~ κ
K_1	0.44	K°	328
O_1	0.42	O°	320
P_1	0.14	P°	329
μ_2		μ°	
S_2	0.02	S°	2
M_2	0.07	M°	358
M_4		M°	
M_6		M°	
N_2	0.01	N°	31

Amplitude relations

(1)	M_4+M_2	
(2)	M_6+M_2	
(3)	S_2+M_2	0.29
(4)	N_2+M_2	0.14
(5)	O_1+K_1	0.95
(6)	K_1+O_1	0.86
(7)	$(K_1+O_1)+M_2$	12.29

Epoch relations

If the direct difference between the constituents of (10) to (13) is more than 180°, add 360° to the lesser one before subtracting or adding. Use the negative sign, if necessary.
Rejection of 720° (but not 360°) admissible in (13) and (14).

(8)	2 M°−M°	
(9)	3 M°−M°	
(10)	S°−M°	4
(11)	M°−N°	−33
(12)	K°−O°	8
(13)	K°+O°	−72
(14)	M°−K°−O°=M°−(13)	430
(15)	MKO=(14)÷2	215

Age of inequalities in hours

(16)	Phase age =0.984×(10)	4
(17)	Parallax age=1.837×(11)	−61
(18)	Diurnal age=0.911×(12)	7

Mean intervals

(19)	v′=Table 1, args. (1) and (8)	
(20)	w′=Table 1, args. (1), (8)±180°	
(21)	(9)−3(19)	
(22)	(9)−3(20)	
(23)	v″=Table 2, args. (2), (21)	
(24)	w″=Table 2, args. (2), (22)	
(25)	v=(19)+(23)	
(26)	w=(20)+(24)	
(27)	M°−(25)	
(28)	M°−(26)±180°	
(29)	HWI=0.0345×(27)	
(30)	LWI=0.0345×(28)	

Mean range

(31)	Cos v=cos (25)	
(32)	Cos w=cos (26)	
(33)	Table 4, arg. (3)	
(34)	Table 5, arg. (7)	
(35)	(31)+(32)+(33)+(34)	
(36)	Cos [(8)−2v]	
(37)	Cos [(8)−2w]	
(38)	Cos [(9)−3v]	
(39)	Cos [(9)−3w]	
(40)	$1.02M_2$×(35)	
(41)	$1.02M_4$[(36)−(37)]	
(42)	$1.02M_6$[(38)+(39)]	
(43)	Mn=(40)+(41)+(42)	

Spring and neap range

(44)	0.536 S_2×(3)	
(45)	$S_2+\mu_2$ cos (2 M°−S°−μ°)	
(46)	1.96−0.08×(7)²	
(47)	(45)×(46)	
(48)	Sg=(43)−(44)+(47)	
(49)	Np=(43)−(44)−(47)	
(50)	Sg+Mn=(48)+(43)	
(51)	Np+Mn=(49)+(43)	

Perigean and apogean range

(52)	Pn+Mn=1+(4)	
(53)	An+Mn=1−0.75×(4)	
(54)	Pn=(43)×(52)	
(55)	An=(43)×(53)	

Tropic intervals

(56)	(15)−½(25)	
(57)	(15)−½(26)−90°	
(58)	Table 9, args. (7), (56)	
(59)	Table 9, args. (7), (57)	
(60)	Table 9a, args. (7), (56)	
(61)	Table 9a, args. (7), (57)	
(62)	TcHHWI=(29)−(58)	
(63)	TcLLWI=(30)−(59)	
(64)	TcLHWI=(29)−(60)	
(65)	TcHLWI=(30)−(61)	

HHWI refers to transit *a* when (56) is in 1st or 4th quadrant, if this interval is approximately M°+29.
LLWI refers to transit *a* when (57) is in 1st or 4th quadrant, if this interval is approximately (M°+180°)+29.

Tropic heights

(66)	Table 11t, args. (7), (56)	
(67)	Table 11t, args. (7), (57)	
(68)	Table 10t, args. (7), (56)	
(69)	Table 10t, args. (7), (57)	
(70)	HWQ=M_2×(66)	
(71)	LWQ=M_2×(67)	
(72)	TcHHW=M_2×(68)	
(73)	TcLLW=−M_2×(69)	
(74)	TcLHW=(72)−(70)	
(75)	TcHLW=(73)+(71)	
(76)	Gc=(72)−(73)	

Mean tide level

(77)	M_4 cos (8)	0.00
(78)	0.03×(6)×(7)× cos (14)	0.11
(79)	MTL=(77)−(78)	−0.11

Mean heights

(80)	Table 13, args. (5), (56)	
(81)	Table 13, args. (5), (57)	
(82)	DHQ=(6)×(80)	
(83)	DLQ=(6)×(81)	
(84)	MHHW=½(43)+(79)+(82)	
(85)	MLLW=−½(43)+(79)−(83)	
(86)	MLHW=(84)−2(82)	
(87)	MHLW=(85)+2(83)	
(88)	Gt=(84)−(85)	

Diurnal tide
when (7) is greater than 4

(89)	R′=reciprocal of (7)	0.08
(90)	MKO±90°=(15)±90°	125
(91)	Table 15, args. (89), (15)	−0.52
(92)	Table 15, args. (89), (90)	0.62
(93)	0.0345 (13)	−2.48
(94)	TcHHWI=(93)−(91)	−1.96
(95)	TcLLWI=(93)−(92)±12.42	9.32
(96)	Table 16, args. (89), (15)	1.04
(97)	Table 16, args. (89), (90)	0.98
(98)	Table 17, args. (89), (15)	0.71
(99)	Table 17, args. (89), (90)	0.69
(100)	TcHHW=(6)×(96)	0.89
(101)	TcLLW=−(6)×(97)	−0.84
(102)	MHHW=(6)×(98)	0.61
(103)	MLLW=−(6)×(99)	−0.59
(104)	Gc=(100)−(101)	1.73
(105)	Gt=(102)−(103)	1.20

Computed by MAW Date 3/22/45 Verified by AG Date 3/22/45

U. S. GOVERNMENT PRINTING OFFICE 815490

GRAPHS SHOWING EFFECTS OF M_4, M_6, AND (K_1+O_1) UPON SEMIDIURNAL WAVE M_2

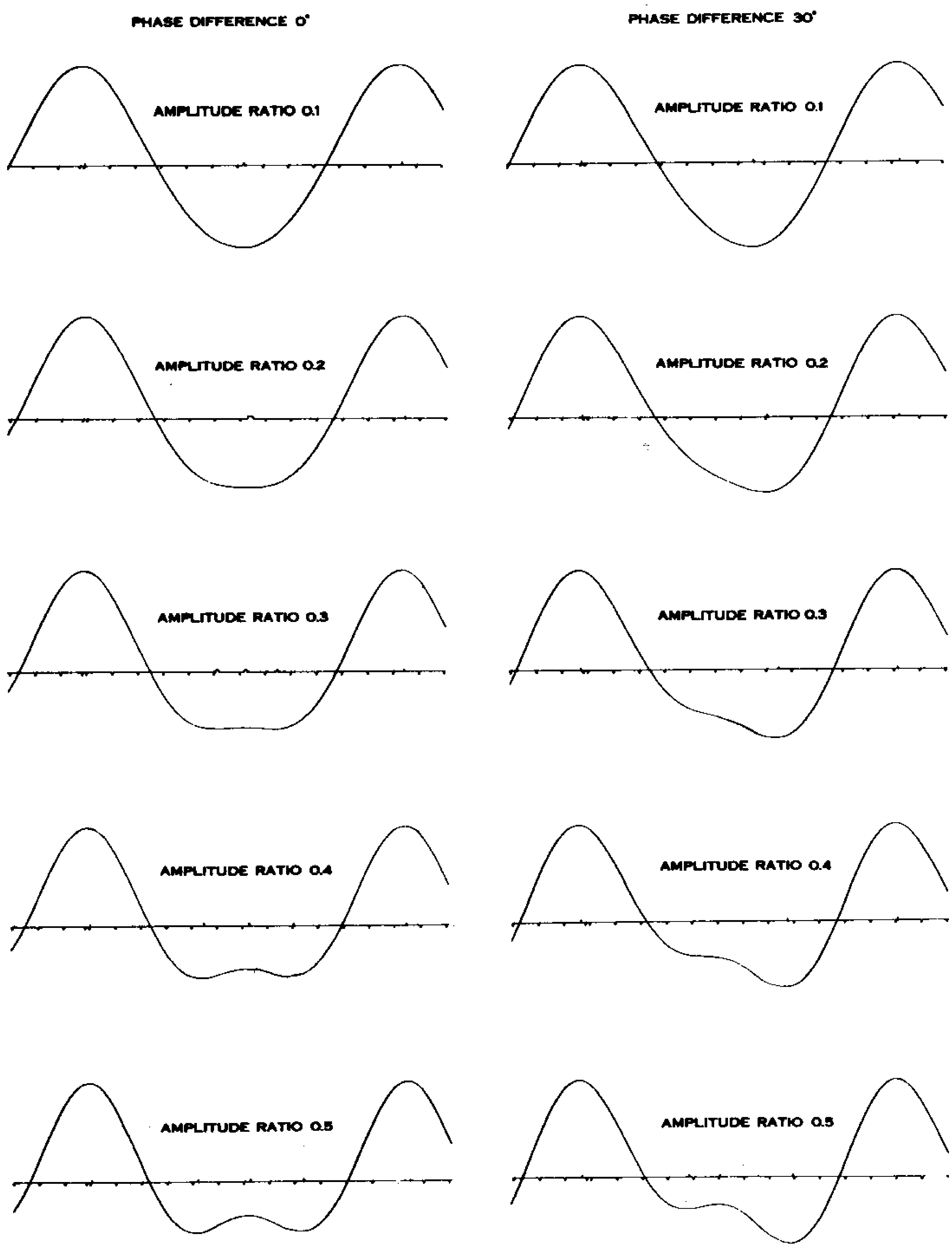
Effect of M_4:- Original curves were traced by predicting-machine with semidiurnal wave M_2 and quarter-diurnal wave M_4 represented by corresponding constituents on machine. Each graph covers a period of approximately 17 solar hours. Expressed in terms of the harmonic constants, the amplitude ratio $= M_4/M_2$ and the phase difference $= 2M_2^\circ - M_4^\circ$. When the amplitude ratio is less than 0.25 the compound wave will be semidiurnal for all phase relations. When the ratio exceeds this amount a double low water will occur with a phase difference of 0° and a double high water with a phase difference of 180°. When the ratio exceeds 0.5, the compound wave is quarter-diurnal for all phase relations.

Effect of M_6:- Original curves were drawn by hand from computations for amplitude and phase relations. Each graph covers 18 lunar hours, the hours marked 0 and 12 corresponding to the unaffected M_2 maximum. Expressed in terms of the harmonic constants, the amplitude ratio $= M_6/M_2$ and the phase difference $= 3M_2^\circ - M_6^\circ$. When the amplitude ratio is less than 1/9 the compound wave will be semidiurnal for all phase relations. When the ratio exceeds this amount a double high water and a double low water will occur with a phase difference of 180°. When the ratio exceeds 1/3 the compound wave will be sixth-diurnal for all phase relations.

Effect of (K_1+O_1):- Original curves were traced by predicting-machine with semidiurnal wave M_2 and diurnal wave (K_1+O_1) represented on the machine by M_4 and M_2, respectively, thus reducing the time scale of the graph by one-half. The period covered by each graph is approximately 27 solar hours with the time marks spaced two hours apart. The amplitude ratio and phase difference of the constituent waves change thoughout the tropical month. At the time of the tropic tides the amplitude ratio $= (K_1+O_1)/M_2$ and the phase difference $= \frac{1}{2}(M_2^\circ - K_1^\circ - O_1^\circ)$. When the amplitude ratio is less than 2.0 the compound wave is semidiurnal for all phase relations. When the ratio exceeds this amount the wave becomes diurnal with phase relations of 45°, 135°, 225° and 315°. When the amplitude ratio exceeds 4.0 the compound wave is diurnal for all phase relations.

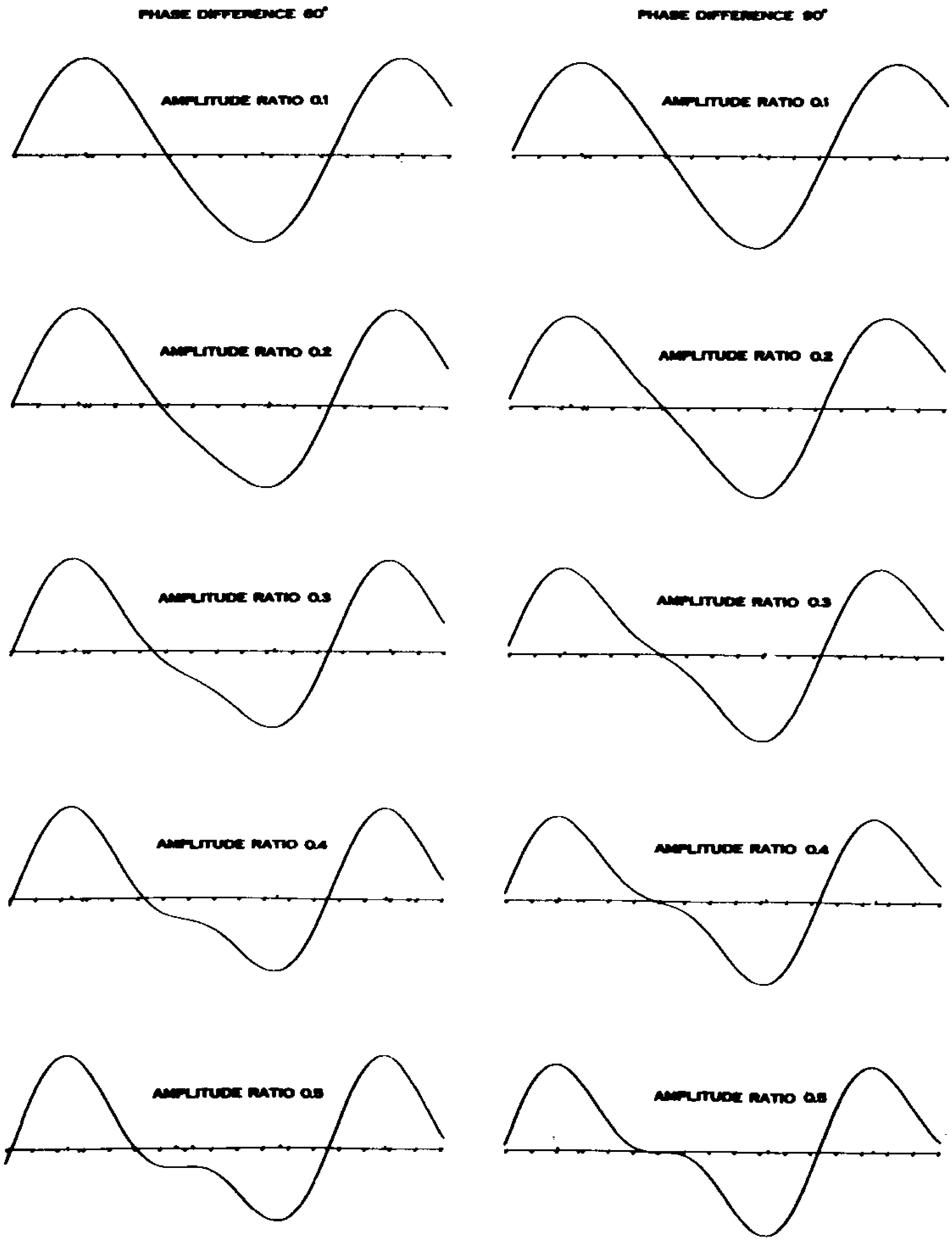
EFFECT OF M_4 UPON M_2

TIME MARKS SPACED 1 HOUR



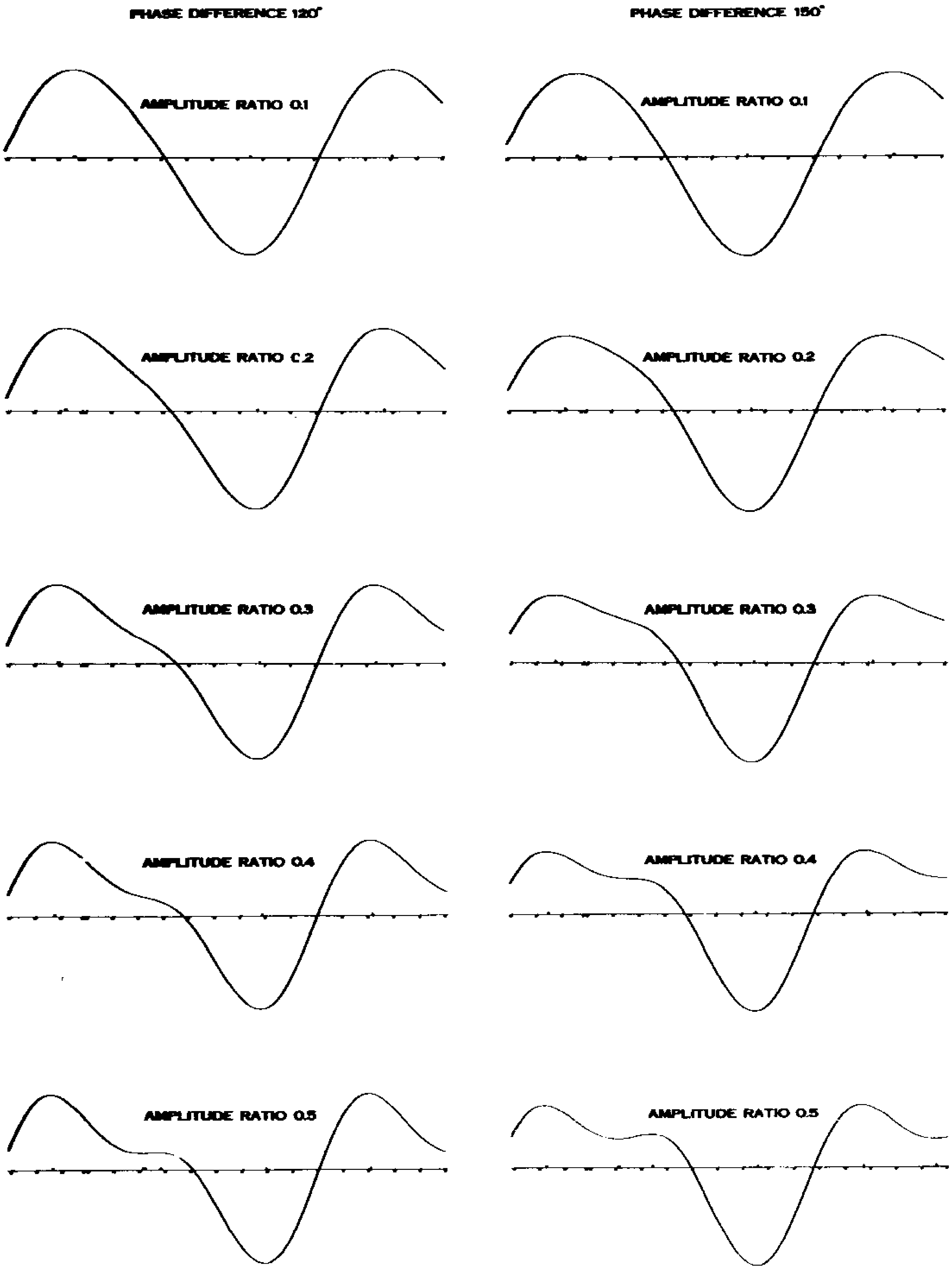
EFFECT OF M_4 UPON M_2

TIME MARKS SPACED 1 HOUR



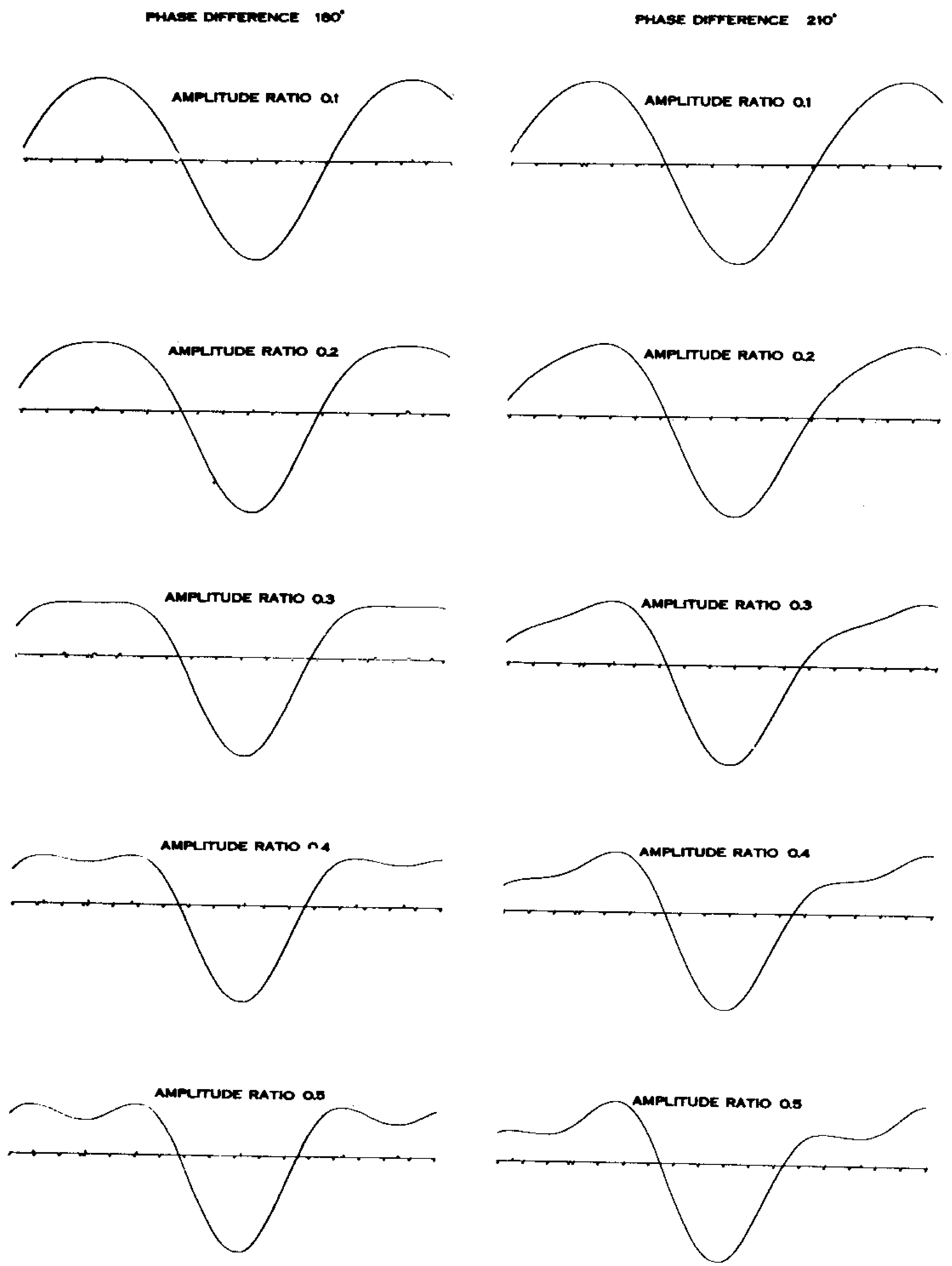
EFFECT OF M_4 UPON M_2

TIME MARKS SPACED 1 HOUR



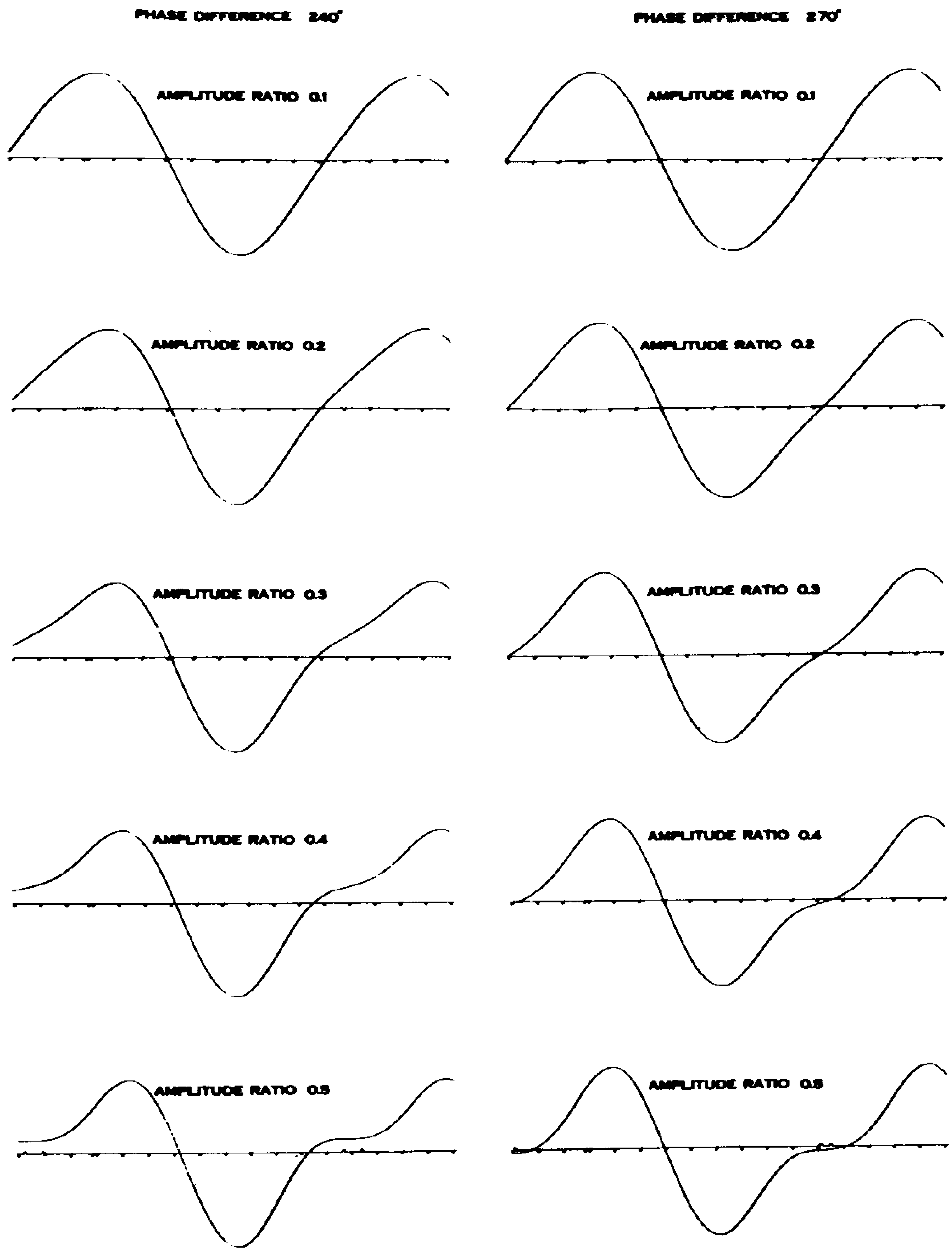
EFFECT OF M_4 UPON M_2

TIME MARKS SPACED 1 HOUR



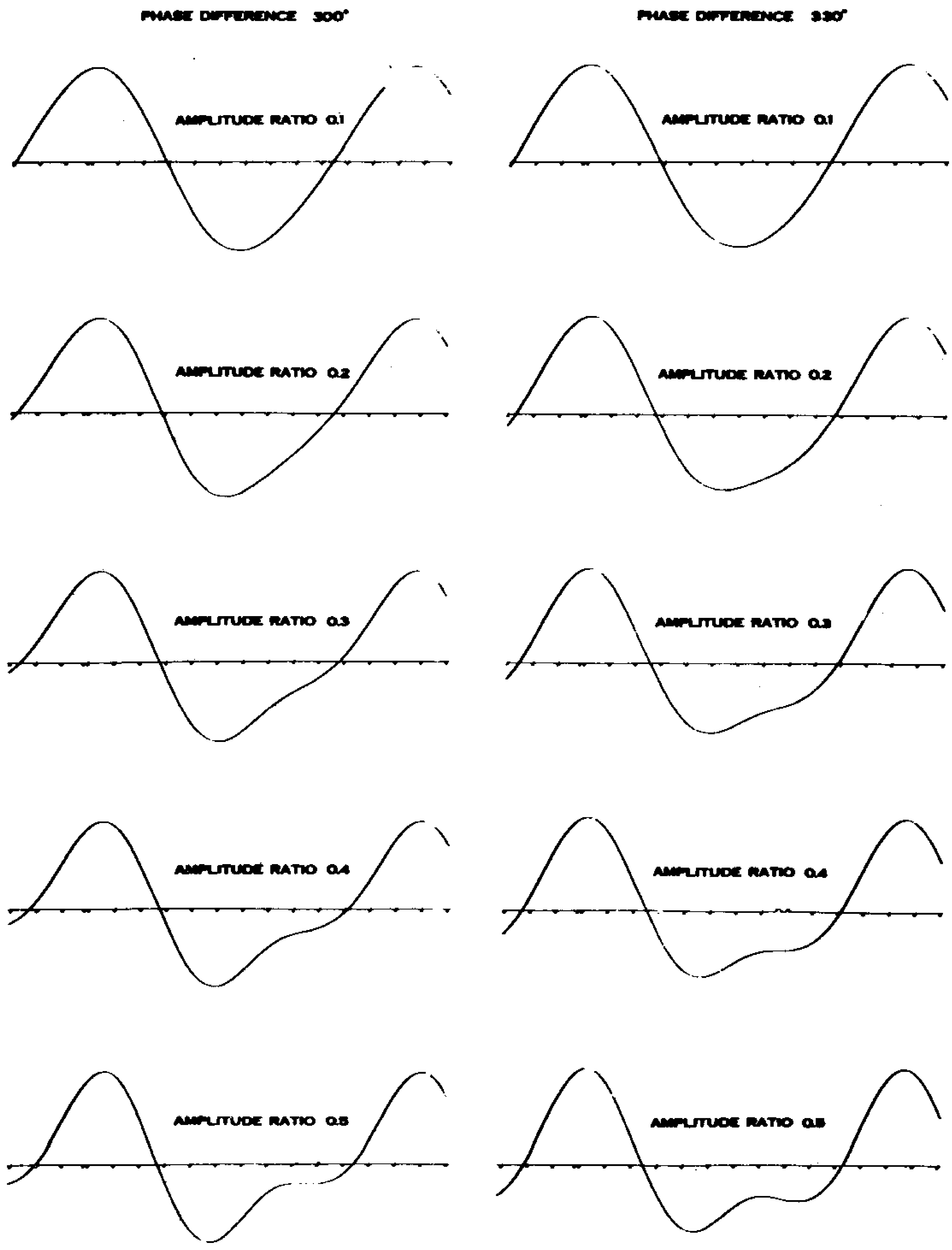
EFFECT OF M_4 UPON M_2

TIME MARKS SPACED 1 HOUR

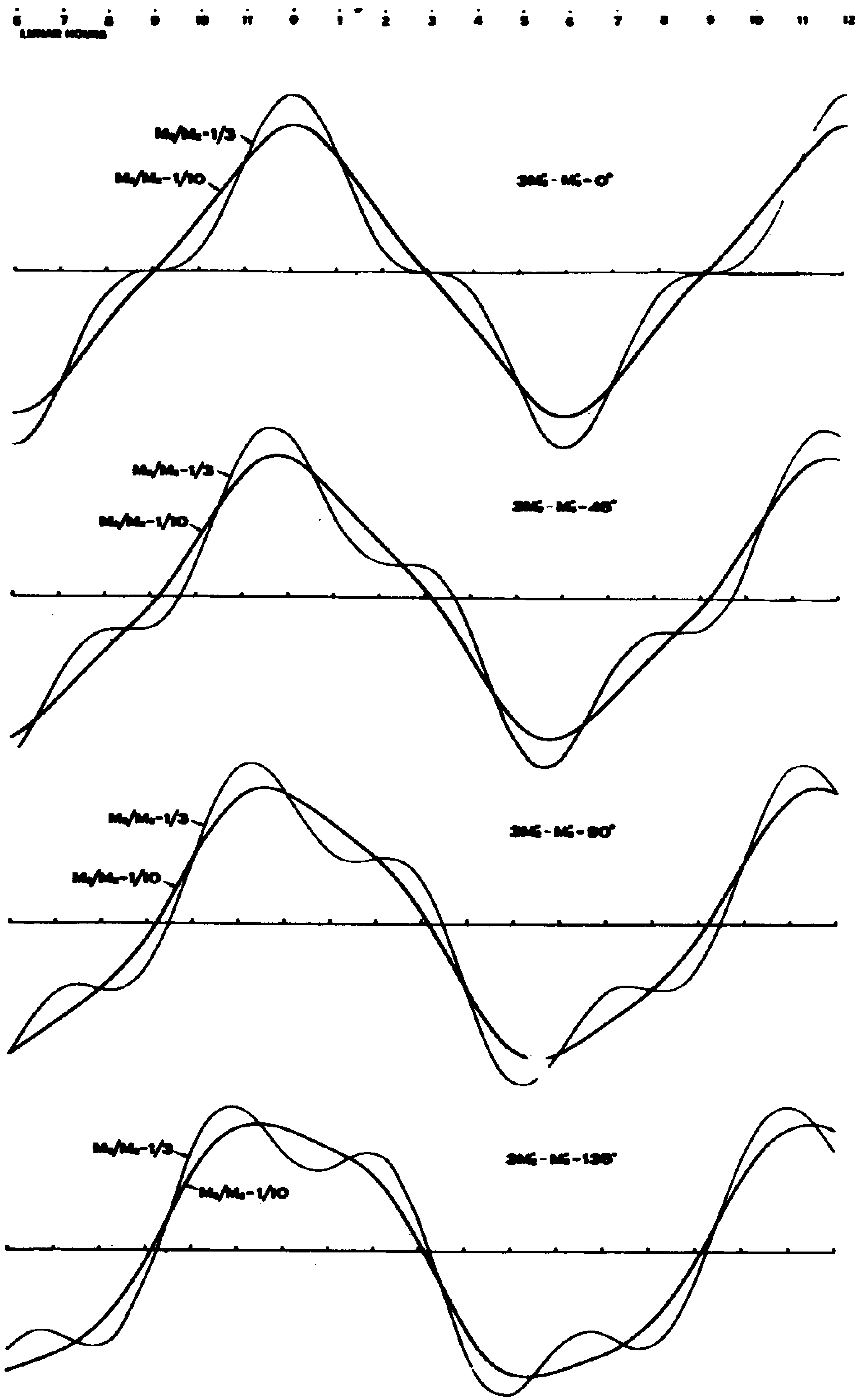


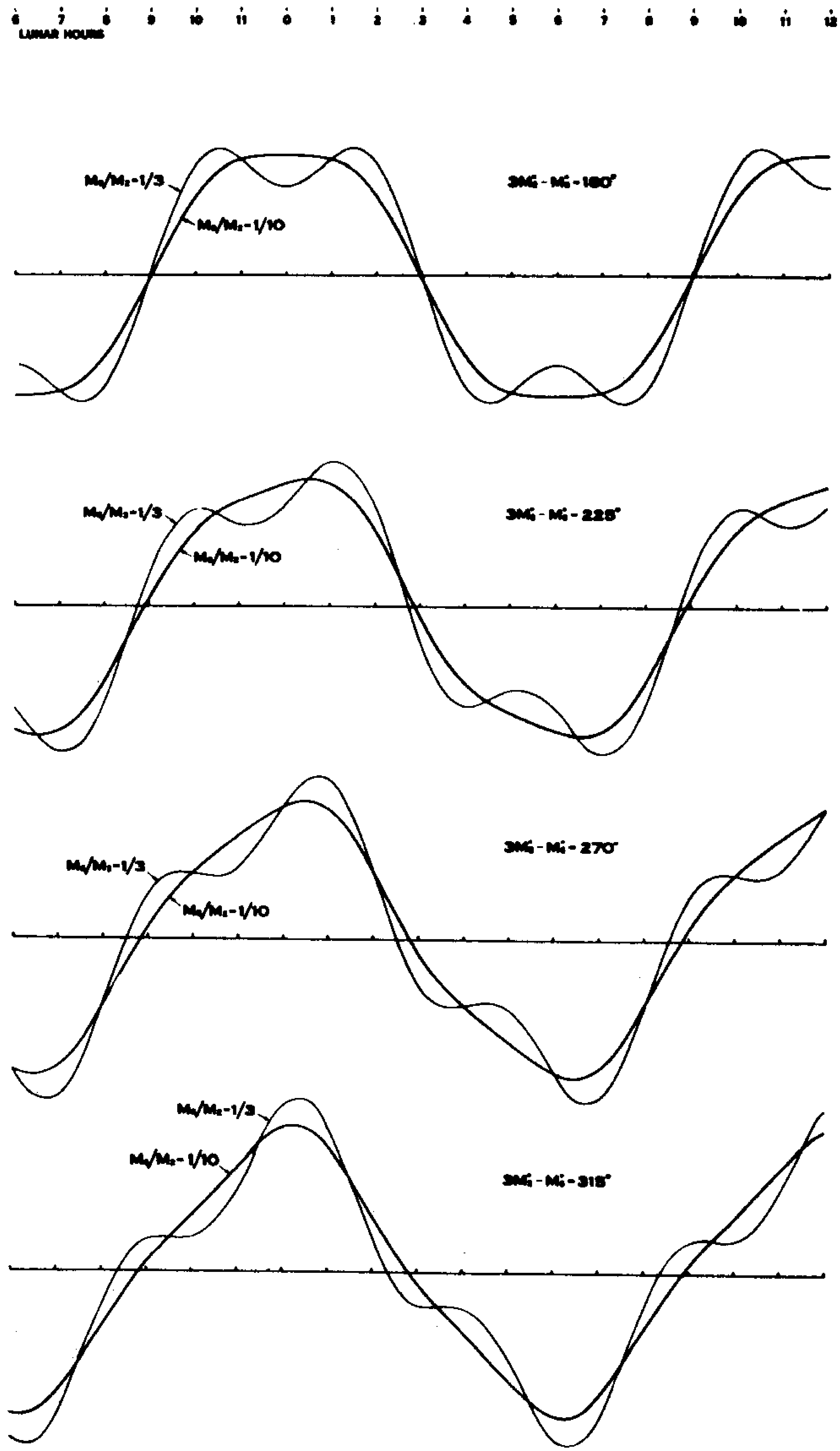
EFFECT OF M_4 UPON M_2

TIME MARKS SPACED 1 HOUR



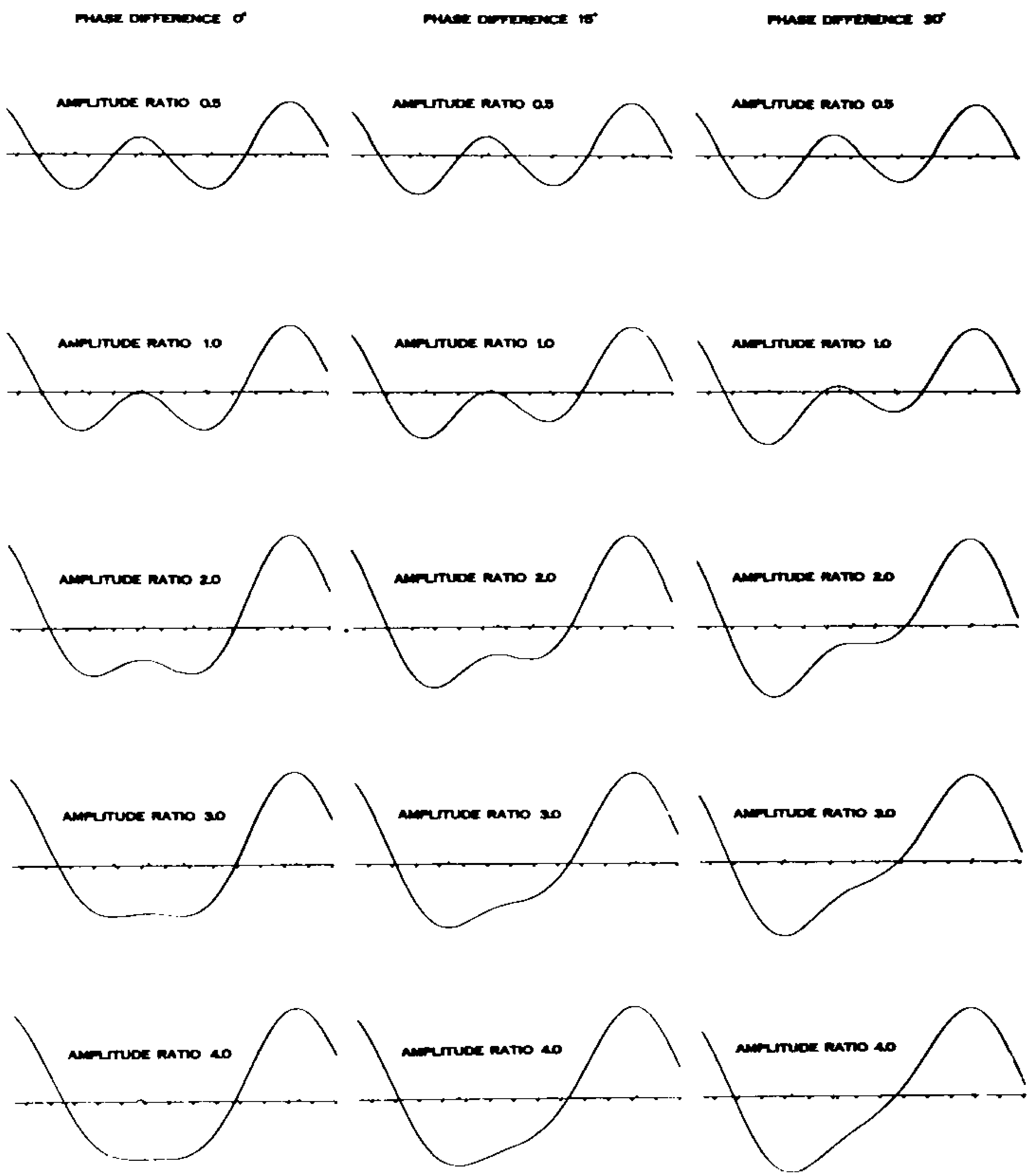
EFFECT OF M_6 UPON M_2




6 7 8 9 10 11 0 1 2 3 4 5 6 7 8 9 10 11 12
LUNAR HOURS
M6/M2 = 1/3
M6/M2 = 1/10
3M2° - M6° = 180°
M6/M2 = 1/3
M6/M2 = 1/10
3M2° - M6° = 225°
M6/M2 = 1/3
M6/M2 = 1/10
3M2° - M6° = 270°
M6/M2 = 1/3
M6/M2 = 1/10
3M2° - M6° = 315°

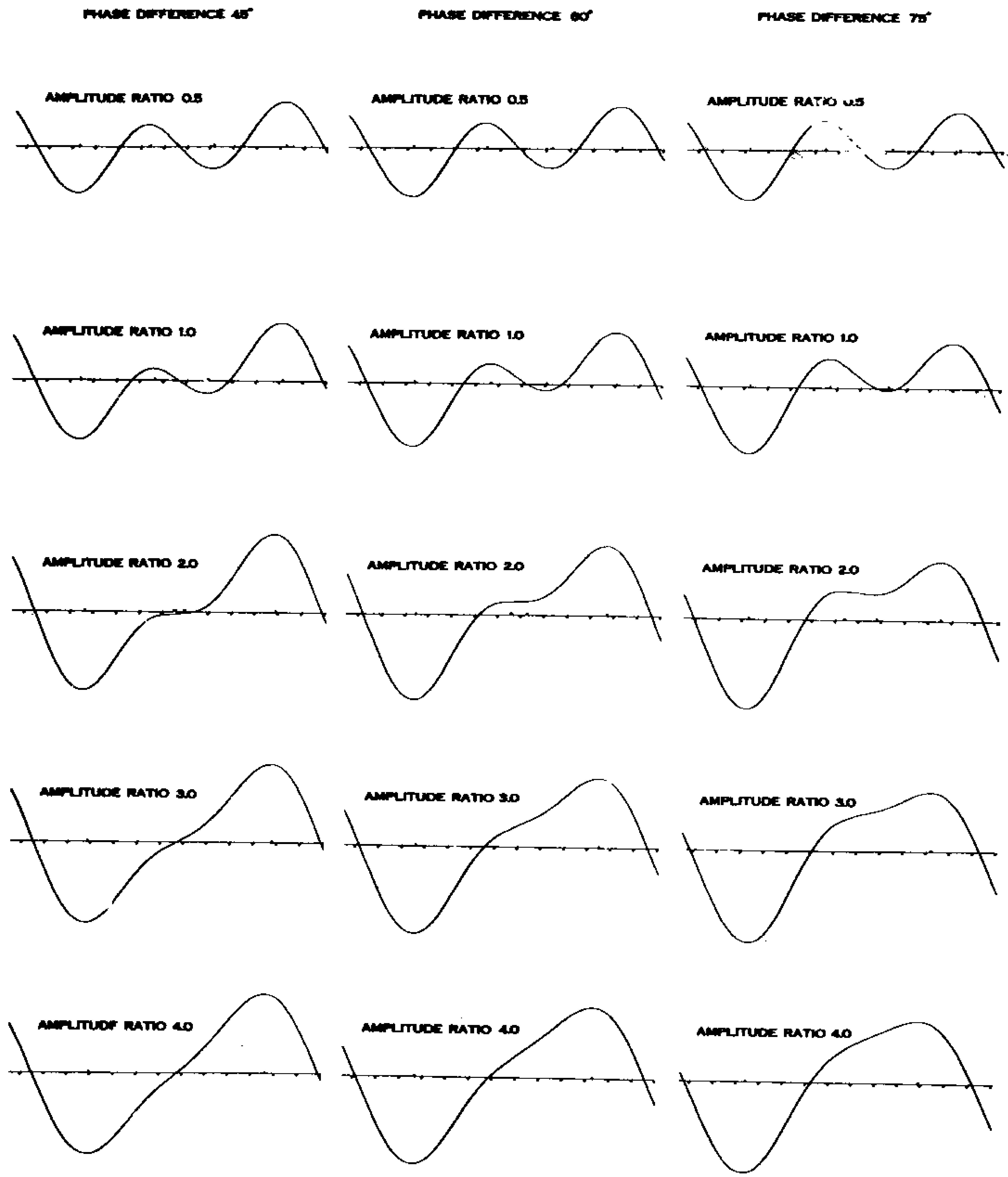
EFFECT OF (K_1+O_1) UPON M_2

TIME MARKS SPACED 2 HOURS



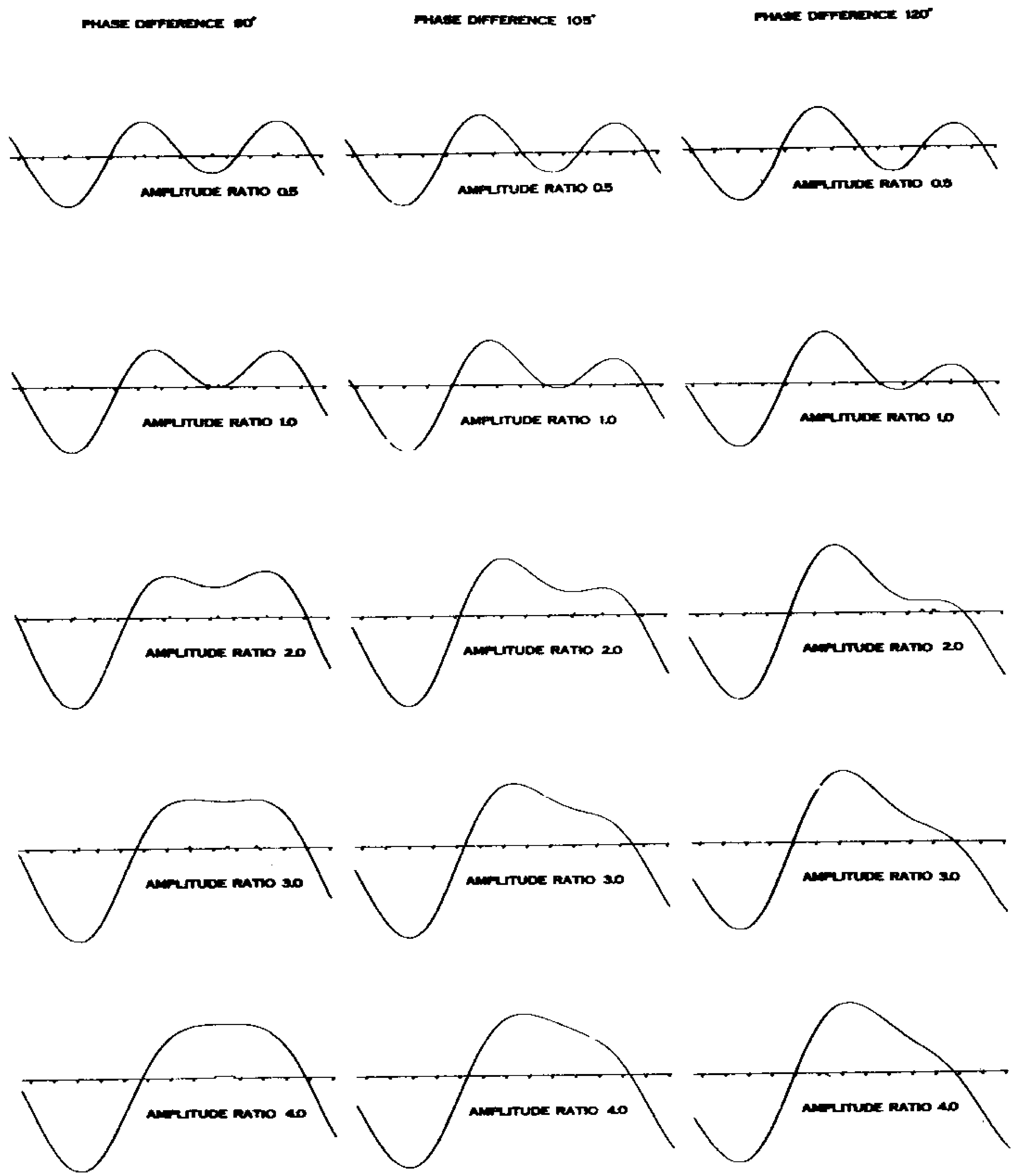
EFFECT OF (K_1+O_1) UPON M_2

TIME MARKS SPACED 2 HOURS



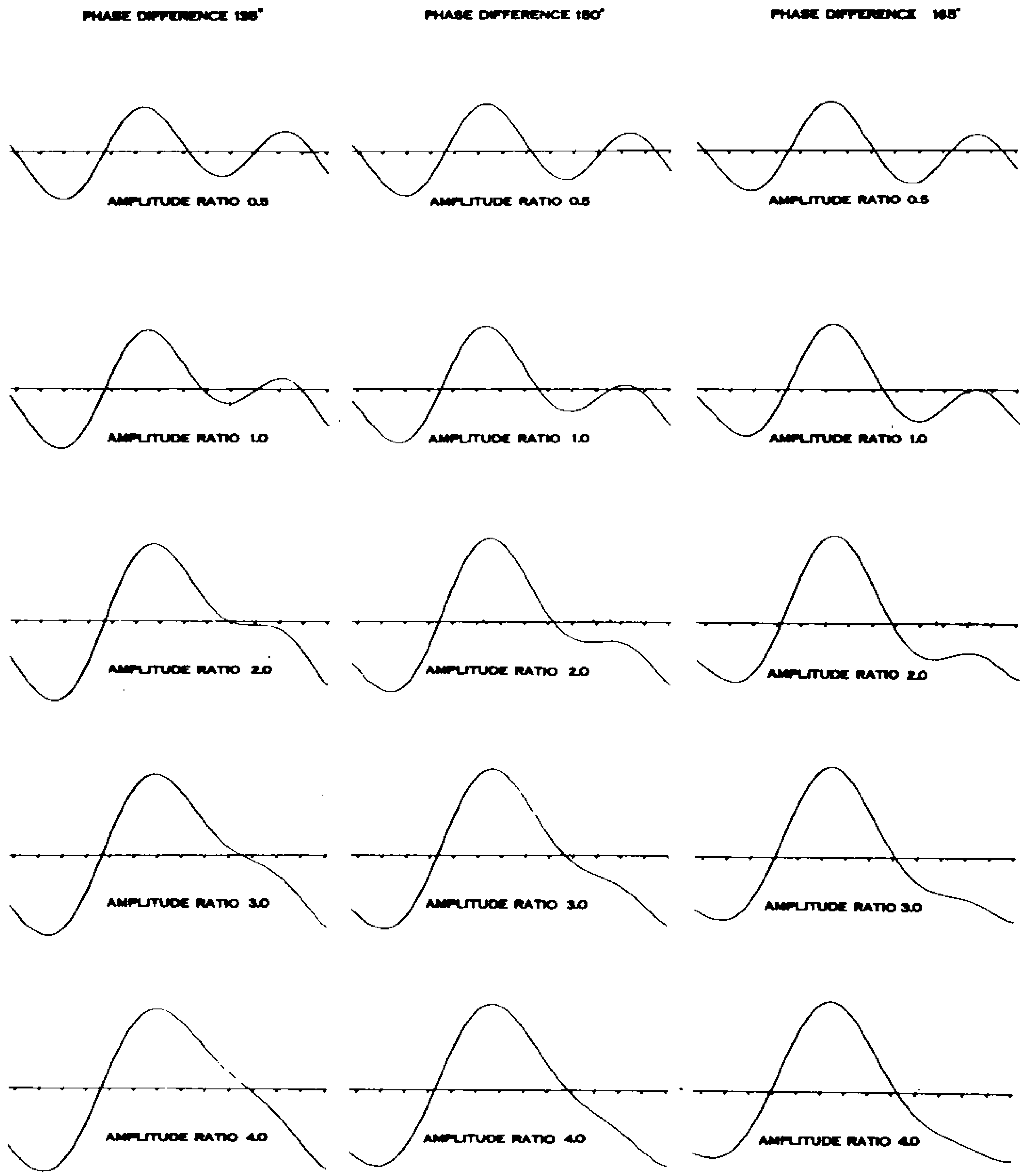
EFFECT OF (K_1+O_1) UPON M_2

TIME MARKS SPACED 2 HOURS



EFFECT OF (K_1+O_1) UPON M_2

TIME MARKS SPACED 2 HOURS



<u>TABLES</u>

*Table used in computing Form 180.

Table 1.- Acceleration in M_2 due to M_4

Phase difference = $2M_2^\circ - M_4^\circ$ for HW and $2M_2^\circ - M_4^\circ \pm 180^\circ$ for LW

Phase diff.	0°:	10°:	20°:	30°:	40°:	50°:	60°:	70°:	80°:	90°:
M_4/M_2	°	°	°	°	°	°	°	°	°	°
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01	0.0	0.2	0.4	0.6	0.7	0.9	1.0	1.1	1.1	1.1
.02	0.0	0.4	0.7	1.1	1.4	1.7	1.9	2.1	2.2	2.3
.03	0.0	0.5	1.1	1.6	2.0	2.4	2.8	3.1	3.3	3.4
.04	0.0	0.7	1.4	2.0	2.6	3.2	3.6	4.0	4.3	4.5
.05	0.0	0.8	1.6	2.4	3.2	3.9	4.5	5.0	5.4	5.6
.06	0.0	1.0	1.9	2.8	3.7	4.5	5.2	5.9	6.4	6.7
.07	0.0	1.1	2.2	3.2	4.2	5.1	6.0	6.7	7.3	7.8
.08	0.0	1.2	2.4	3.6	4.7	5.7	6.7	7.5	8.2	8.8
.09	0.0	1.3	2.6	3.9	5.1	6.3	7.4	8.3	9.1	9.8
.10	0.0	1.4	2.8	4.2	5.5	6.8	8.0	9.1	10.0	10.7
.11	0.0	1.5	3.0	4.5	5.9	7.3	8.6	9.8	10.8	11.7
.12	0.0	1.6	3.2	4.8	6.3	7.8	9.2	10.5	11.6	12.6
.13	0.0	1.7	3.4	5.1	6.7	8.3	9.7	11.1	12.3	13.4
.14	0.0	1.8	3.6	5.3	7.0	8.7	10.3	11.7	13.1	14.2
.15	0.0	1.9	3.7	5.6	7.4	9.1	10.8	12.3	13.8	15.0
.16	0.0	1.9	3.9	5.8	7.7	9.5	11.2	12.9	14.4	15.8
.17	0.0	2.0	4.0	6.0	8.0	9.9	11.7	13.4	15.1	16.5
.18	0.0	2.1	4.2	6.2	8.3	10.2	12.1	14.0	15.7	17.3
.19	0.0	2.2	4.3	6.4	8.5	10.6	12.6	14.5	16.3	17.9
.20	0.0	2.2	4.4	6.6	8.8	10.9	13.0	14.9	16.8	18.6
.21	0.0	2.3	4.6	6.8	9.0	11.2	13.3	15.4	17.4	19.2
.22	0.0	2.3	4.7	7.0	9.3	11.5	13.7	15.8	17.9	19.8
.23	0.0	2.4	4.8	7.2	9.5	11.8	14.1	16.3	18.4	20.4
.24	0.0	2.4	4.9	7.3	9.7	12.1	14.4	16.7	18.8	20.9
.25	0.0	2.5	5.0	7.5	9.9	12.3	14.7	17.0	19.3	21.5
.26	0.0	2.5	5.1	7.6	10.1	12.6	15.0	17.4	19.7	22.0
.27	0.0	2.6	5.2	7.8	10.3	12.8	15.3	17.7	20.2	22.5
.28	0.0	2.6	5.3	7.9	10.5	13.1	15.6	18.1	20.6	22.9
.29	0.0	2.7	5.4	8.0	10.7	13.3	15.9	18.5	21.0	23.4
.30	0.0	2.7	5.4	8.2	10.9	13.5	16.2	18.8	21.3	23.8
Phase diff.	360°:	350°:	340°:	330°:	320°:	310°:	300°:	290°:	280°:	270°:

Tabular values positive with top arguments, negative with bottom arguments. Further explanation in text.

Table 1.- Acceleration in M_2 due to M_4 (Continued)

Phase difference = $2M_2^\circ - M_4^\circ$ for HW and $2M_2^\circ - M_4^\circ \pm 180^\circ$ for LW

Phase diff.	90°:	100°:	110°:	120°:	130°:	140°:	150°:	160°:	170°:	180°:
M_4/M_2	°	°	°	°	°	°	°	°	°	°
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01	1.1	1.1	1.1	1.0	0.9	0.8	0.6	0.4	0.2	0.0
.02	2.3	2.3	2.2	2.1	1.8	1.6	1.2	0.8	0.4	0.0
.03	3.4	3.4	3.3	3.1	2.8	2.4	1.9	1.3	0.7	0.0
.04	4.5	4.6	4.5	4.3	3.9	3.3	2.6	1.8	0.9	0.0
.05	5.6	5.7	5.7	5.4	5.0	4.3	3.4	2.4	1.2	0.0
.06	6.7	6.9	6.9	6.6	6.1	5.3	4.3	3.0	1.6	0.0
.07	7.8	8.0	8.0	7.8	7.3	6.4	5.2	3.7	1.9	0.0
.08	8.8	9.1	9.2	9.0	8.5	7.5	6.2	4.4	2.3	0.0
.09	9.8	10.2	10.4	10.2	9.7	8.7	7.2	5.2	2.8	0.0
.10	10.7	11.3	11.5	11.4	11.0	10.0	8.4	6.1	3.3	0.0
.11	11.7	12.3	12.7	12.7	12.2	11.3	9.6	7.1	3.8	0.0
.12	12.6	13.3	13.8	13.9	13.5	12.6	10.9	8.2	4.5	0.0
.13	13.4	14.3	14.8	15.1	14.8	13.9	12.2	9.4	5.2	0.0
.14	14.2	15.2	15.9	16.2	16.1	15.3	13.6	10.6	6.0	0.0
.15	15.0	16.1	16.9	17.4	17.4	16.7	15.1	12.1	7.0	0.0
.16	15.8	17.0	17.9	18.5	18.6	18.1	16.6	13.6	8.2	0.0
.17	16.5	17.8	18.9	19.6	19.9	19.5	18.1	15.2	9.5	0.0
.18	17.3	18.7	19.8	20.7	21.1	20.9	19.7	16.9	11.0	0.0
.19	17.9	19.4	20.7	21.7	22.3	22.2	21.2	18.6	12.7	0.0
.20	18.6	20.2	21.6	22.7	23.4	23.5	22.8	20.5	14.7	0.0
.21	19.2	20.9	22.4	23.6	24.5	24.8	24.3	22.3	16.9	0.0
.22	19.8	21.6	23.2	24.6	25.6	26.1	25.8	24.1	19.2	0.0
.23	20.4	22.3	24.0	25.5	26.6	27.3	27.3	25.9	21.6	0.0
.24	20.9	22.9	24.7	26.3	27.6	28.5	28.7	27.7	24.0	0.0
.25	21.5	23.5	25.4	27.1	28.5	29.6	30.0	29.4	26.4	0.0
.26	22.0	24.1	26.1	27.9	29.5	30.7	31.3	31.0	28.7	15.9
.27	22.5	24.7	26.8	28.7	30.3	31.7	32.5	32.5	30.8	22.2
.28	22.9	25.2	27.4	29.4	31.2	32.7	33.7	34.0	32.9	26.8
.29	23.4	25.7	28.0	30.1	32.0	33.6	34.9	35.4	34.8	30.5
.30	23.8	26.3	28.6	30.8	32.8	34.5	36.0	36.8	36.6	33.6
Phase diff.	270°:	260°:	250°:	240°:	230°:	220°:	210°:	200°:	190°:	180°:

Tabular values positive with top arguments, negative with bottom arguments. Further explanation in text.

Table 2.- Acceleration in M_2 due to M_6

Phase difference = $3M_2^\circ - M_6^\circ - 3v'$ for HW and $3M_2^\circ - M_6^\circ - 3w'$ for LW

Phase diff.	0°:	10°:	20°:	30°:	40°:	50°:	60°:	70°:	80°:	90°:
M_6/M_2	°	°	°	°	°	°	°	°	°	°
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01	0.0	0.3	0.5	0.8	1.0	1.2	1.4	1.6	1.7	1.7
.02	0.0	0.5	1.0	1.5	1.9	2.3	2.7	3.0	3.2	3.4
.03	0.0	0.7	1.4	2.1	2.7	3.3	3.9	4.3	4.7	5.0
.04	0.0	0.9	1.8	2.6	3.4	4.2	4.9	5.5	6.1	6.5
.05	0.0	1.0	2.1	3.1	4.0	5.0	5.8	6.6	7.3	7.9
.06	0.0	1.2	2.3	3.5	4.6	5.6	6.7	7.6	8.4	9.2
.07	0.0	1.3	2.6	3.8	5.1	6.3	7.4	8.5	9.5	10.4
.08	0.0	1.4	2.8	4.2	5.5	6.8	8.1	9.3	10.4	11.4
.09	0.0	1.5	3.0	4.4	5.9	7.3	8.7	10.0	11.2	12.4
.10	0.0	1.6	3.1	4.7	6.2	7.8	9.2	10.7	12.0	13.3
.11	0.0	1.7	3.3	4.9	6.6	8.2	9.7	11.3	12.7	14.1
.12	0.0	1.7	3.5	5.2	6.9	8.5	10.2	11.8	13.4	14.9
.13	0.0	1.8	3.6	5.4	7.1	8.9	10.6	12.3	13.9	15.5
.14	0.0	1.9	3.7	5.6	7.4	9.2	11.0	12.8	14.5	16.2
.15	0.0	1.9	3.8	5.7	7.6	9.5	11.4	13.2	15.0	16.7
.16	0.0	2.0	3.9	5.9	7.8	9.8	11.7	13.6	15.4	17.3
.17	0.0	2.0	4.0	6.0	8.0	10.0	12.0	13.9	15.9	17.8
.18	0.0	2.1	4.1	6.2	8.2	10.3	12.3	14.3	16.3	18.2
.19	0.0	2.1	4.2	6.3	8.4	10.5	12.5	14.6	16.6	18.6
.20	0.0	2.1	4.3	6.4	8.6	10.7	12.8	14.9	17.0	19.0
.21	0.0	2.2	4.4	6.5	8.7	10.9	13.0	15.2	17.3	19.4
.22	0.0	2.2	4.4	6.6	8.8	11.0	13.2	15.4	17.6	19.7
.23	0.0	2.2	4.5	6.7	9.0	11.2	13.4	15.7	17.9	20.1
.24	0.0	2.3	4.6	6.8	9.1	11.4	13.6	15.9	18.1	20.4
.25	0.0	2.3	4.6	6.9	9.2	11.5	13.8	16.1	18.4	20.7
.26	0.0	2.3	4.7	7.0	9.3	11.7	14.0	16.3	18.6	20.9
.27	0.0	2.4	4.7	7.1	9.4	11.8	14.1	16.5	18.8	21.2
.28	0.0	2.4	4.8	7.2	9.5	11.9	14.3	16.7	19.0	21.4
.29	0.0	2.4	4.8	7.2	9.6	12.0	14.4	16.8	19.2	21.6
.30	0.0	2.4	4.9	7.3	9.7	12.2	14.6	17.0	19.4	21.9
.31	0.0	2.5	4.9	7.4	9.8	12.3	14.7	17.2	19.6	22.1
.32	0.0	2.5	4.9	7.4	9.9	12.4	14.8	17.3	19.8	22.3
.33	0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	19.9	22.4
Phase diff.	360°:	350°:	340°:	330°:	320°:	310°:	300°:	290°:	280°:	270°:

Tabular values positive with top arguments, negative with bottom arguments. Further explanation in text.

Table 2.- Acceleration in M_2 due to M_6 (Continued)

Phase difference = $3M_2^\circ - M_6^\circ - 3v'$ for HW and $3M_6^\circ - M_6^\circ - 3w'$ for LW

Phase diff.	90°:	100°:	110°:	120°:	130°:	140°:	150°:	160°:	170°:	180°:
M_6/M_2	°	°	°	°	°	°	°	°	°	°
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01	1.7	1.7	1.7	1.6	1.4	1.2	0.9	0.6	0.3	0.0
.02	3.4	3.4	3.4	3.2	2.9	2.5	2.0	1.4	0.7	0.0
.03	5.0	5.1	5.1	5.0	4.6	4.1	3.3	2.3	1.2	0.0
.04	6.5	6.8	6.9	6.8	6.4	5.8	4.8	3.5	1.8	0.0
.05	7.9	8.3	8.6	8.6	8.3	7.7	6.6	4.9	2.6	0.0
.06	9.2	9.8	10.2	10.4	10.2	9.7	8.5	6.6	3.7	0.0
.07	10.4	11.1	11.7	12.1	12.1	11.7	10.7	8.7	5.2	0.0
.08	11.4	12.3	13.1	13.6	13.9	13.7	12.9	11.1	7.3	0.0
.09	12.4	13.5	14.4	15.1	15.6	15.6	15.1	13.7	10.0	0.0
.10	13.3	14.5	15.6	16.4	17.1	17.4	17.3	16.2	13.2	0.0
.11	14.1	15.4	16.6	17.7	18.5	19.1	19.3	18.7	16.5	0.0
.12	14.9	16.3	17.6	18.8	19.8	20.6	21.1	20.9	19.6	13.6
.13	15.5	17.1	18.5	19.8	21.0	22.0	22.7	22.9	22.3	19.3
.14	16.2	17.8	19.3	20.8	22.1	23.3	24.2	24.8	24.7	23.2
.15	16.7	18.4	20.1	21.6	23.1	24.4	25.5	26.4	26.7	26.2
.16	17.3	19.1	20.8	22.4	24.0	25.5	26.8	27.8	28.5	28.6
.17	17.8	19.6	21.4	23.2	24.8	26.4	27.9	29.1	30.1	30.6
.18	18.2	20.1	22.0	23.8	25.6	27.3	28.9	30.3	31.5	32.4
.19	18.6	20.6	22.6	24.5	26.3	28.1	29.8	31.4	32.8	33.9
.20	19.0	21.1	23.1	25.0	27.0	28.8	30.6	32.3	33.9	35.3
.21	19.4	21.5	23.5	25.6	27.6	29.5	31.4	33.2	34.9	36.5
.22	19.7	21.9	24.0	26.1	28.1	30.1	32.1	34.0	35.8	37.5
.23	20.1	22.2	24.4	26.5	28.7	30.7	32.8	34.8	36.7	38.5
.24	20.4	22.6	24.8	27.0	29.1	31.3	33.4	35.4	37.5	39.4
.25	20.7	22.9	25.2	27.4	29.6	31.8	34.0	36.1	38.2	40.2
.26	20.9	23.2	25.5	27.8	30.0	32.3	34.5	36.7	38.8	40.9
.27	21.2	23.5	25.8	28.1	30.4	32.7	35.0	37.2	39.4	41.6
.28	21.4	23.8	26.1	28.5	30.8	33.1	35.4	37.7	40.0	42.3
.29	21.6	24.0	26.4	28.8	31.2	33.5	35.9	38.2	40.5	42.9
.30	21.9	24.3	26.7	29.1	31.5	33.9	36.3	38.7	41.0	43.4
.31	22.1	24.5	26.9	29.4	31.8	34.3	36.7	39.1	41.5	43.9
.32	22.3	24.7	27.2	29.7	32.1	34.6	37.0	39.5	42.0	44.4
.33	22.4	24.9	27.4	29.9	32.4	34.9	37.4	39.9	42.4	44.9
Phase diff.	270°:	260°:	250°:	240°:	230°:	220°:	210°:	200°:	190°:	180°:

Tabular values positive with top arguments, negative with bottom arguments. Further explanation in text.

Wave $M_2 + K_1 + O_1$			Wave $M_2 + M_4$			Wave $M_2 + M_6$			
P	R	at	P	R	at	P	R	at	at
°		°	°		°	°		°	°
0	4.00	360.0	0	0.25	180.0	0	0.33	90.0	270.0
15	2.46	425.6	15	0.35	145.5	15	0.33	84.4	264.4
30	2.10	439.6	30	0.41	132.2	30	0.33	78.7	258.7
45	2.00	450.0	45	0.45	120.8	45	0.32	73.1	253.1
60	2.10	460.4	60	0.48	110.2	60	0.31	67.4	247.4
75	2.46	474.4	75	0.49	100.0	75	0.30	61.6	241.6
90	4.00	540.0	90	0.50	90.0	90	0.28	55.7	235.7
105	2.46	605.6	105	0.49	80.0	105	0.26	49.7	229.7
120	2.10	619.6	120	0.48	69.8	120	0.24	43.6	223.6
135	2.00	630.0	135	0.45	59.2	135	0.22	37.1	217.1
150	2.10	640.4	150	0.41	47.8	150	0.19	30.0	210.0
165	2.46	654.4	165	0.35	34.5	165	0.16	21.7	201.7
180	4.00	0.0	180	0.25	0.0	180	0.11	0.0	180.0
195	2.46	65.6	195	0.35	325.5	195	0.16	338.3	158.3
210	2.10	79.6	210	0.41	312.2	210	0.19	330.0	150.0
225	2.00	90.0	225	0.45	300.8	225	0.22	322.9	142.9
240	2.10	100.4	240	0.48	290.2	240	0.24	316.4	136.4
255	2.46	114.4	255	0.49	280.0	255	0.26	310.3	130.3
270	4.00	180.0	270	0.50	270.0	270	0.28	304.3	124.3
285	2.46	245.6	285	0.49	260.0	285	0.30	298.4	118.4
300	2.10	259.6	300	0.48	249.8	300	0.31	292.6	112.6
315	2.00	270.0	315	0.45	239.2	315	0.32	286.9	106.9
330	2.10	280.4	330	0.41	227.8	330	0.33	281.3	101.3
345	2.46	294.4	345	0.35	214.5	345	0.33	275.6	95.6
360	4.00	360.0	360	0.25	180.0	360	0.33	270.0	90.0
$P = \frac{1}{2}(M_2^\circ - K^\circ - O_1^\circ)$			$P = 2M_2^\circ - M_4^\circ$			$P = 3M_2^\circ - M_6^\circ$			
$R = (K_1 + O_1)/M_2$			$R = M_4/M_2$			$R = M_6/M_2$			

In each compound wave the principal constituent is M_2 and the wave remains semidiurnal until the ratio "R" exceeds the critical value given for the phase relation. When this limit is exceeded the first wave becomes diurnal, the second quarter-diurnal, and the last sixth-diurnal. The critical points in the compound wave at which the extra tides disappear or reappear are indicated by the "at" values, which are expressed in semidiurnal degrees and are reckoned from the M_2 maximum. The entire period of the $(M_2+K_1+O_1)$ wave covers two periods or 720° of the M_2 constituent, and in this case the "at" is reckoned from the first M_2 maximum following the moon's "a" transit.

 Table 4.—Effect of semidiurnal constituents on range of tide.

S_2/M_2	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.020	0.020	0.020	0.021	0.021	0.021	0.022	0.023	0.024	0.025
0.1	.026	.027	.028	.030	.031	.033	.035	.037	.039	.041
0.2	.043	.045	.048	.051	.053	.056	.059	.062	.065	.069
0.3	.072	.075	.079	.083	.087	.091	.095	.099	.103	.108
0.4	.112	.117	.122	.127	.132	.137	.142	.147	.153	.159
0.5	.164	.170	.176	.182	.188	.195	.201	.207	.214	.221
0.6	.228	.235	.242	.249	.256	.264	.271	.279	.287	.295
0.7	.303	.311	.319	.327	.336	.345	.353	.362	.371	.380
0.8	.389	.399	.408	.417	.427	.437	.447	.457	.467	.477
0.9	.487	.498	.508	.519	.530	.541	.552	.563	.574	.586

Table = $0.020 + 0.577\ (S_2/M_2)^2$

$\frac{K_1+O_1}{M_2}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
0.1	.001	.001	.001	.001	.001	.002	.002	.002	.002	.003
0.2	.003	.003	.003	.004	.004	.004	.005	.005	.006	.006
0.3	.006	.007	.007	.008	.008	.009	.009	010	.010	.011
0.4	.012	.012	.013	.013	.014	.015	.015	.016	.017	.017
0.5	.018	.019	.019	.020	.021	.022	.023	.023	.024	.025
0.6	.026	.027	.028	.029	.029	.030	.031	.032	.033	.034
0.7	.035	.036	.037	.038	.039	.040	.042	.043	.044	.045
0.8	.046	.047	.048	.050	.051	.052	.053	.054	.056	.057
0.9	.058	.060	.061	.062	.064	.065	.066	.068	.069	.071
1.0	.072	.073	.075	.076	.078	.079	.081	.082	.084	.086
1.1	.087	.089	.090	.092	.094	.095	.097	.099	.100	.102
1.2	.104	.105	.107	.109	.111	.112	.114	.116	.118	.120
1.3	.122	.124	.125	.127	.129	.131	.133	.135	.137	.139
1.4	.141	.143	.145	.147	.149	.151	.153	.156	.158	.160
1.5	.162	.164	.166	.169	.171	.173	.175	.177	.180	.182
1.6	.184	.187	.189	.191	.194	.196	.198	.201	.203	.206
1.7	.208	.211	.213	.215	.218	.220	.223	.226	.228	.231
1.8	.233	.236	.238	.241	.244	.246	.249	.252	.254	.257
1.9	.260	.263	.265	.268	.271	.274	.277	.279	.282	.285
2.0	.288	.291	.294	.297	.300	.303	.306	.309	.312	.315
2.1	.318	.321	.324	.327	.330	.333	.336	.339	.342	.345
2.2	.348	.352	.355	.358	.361	.365	.368	.371	.374	.378
2.3	.381	.384	.388	.391	.394	.398	.401	.404	.408	.411
2.4	.415	.418	.422	.425	.429	.432	.436	.439	.443	.446
2.5	.450	.454	.457	.461	.465	.468	.472	.476	.479	.483
2.6	.487	.490	.494	.498	.502	.506	.509	.513	.517	.521
2.7	.525	.529	.533	.537	.541	.544	.548	.552	.556	.560
2.8	.564	.569	.573	.577	.581	.585	.589	.593	.597	.601
2.9	.606	.610	.614	.618	.622	.627	.631	.635	.639	.644
3.0	.648	.652	.657	.661	.665	.670	.674	.679	.683	.687

Table = $0.072\ (K_1+O_1)^2/M_2^2$

Table 6.- Values of C in Formula (60)

ø / r	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	0.98	0.96	0.93	0.90	0.87	0.84	0.82	0.82
0.2	1.00	0.99	.97	.93	.88	.82	.76	.71	.68	.67
0.3	1.00	.99	.96	.91	.84	.76	.68	.61	.56	.54
0.4	1.00	.99	.95	.89	.81	.72	.62	.53	.46	.43
0.5	1.00	.99	.95	.88	.80	.69	.58	.46	.37	.33
0.6	1.00	.99	.94	.88	.78	.67	.54	.41	.30	.25
0.7	1.00	.99	.94	.87	.77	.66	.52	.38	.25	.18
0.8	1.00	.98	.94	.87	.77	.65	.51	.36	.21	.11
0.9	1.00	.98	.94	.87	.77	.64	.50	.35	.18	.05
1.0	1.00	.98	.94	.87	.77	.64	.50	.34	.17	.00
1.1	1.00	.98	.94	.87	.77	.64	.50	.34	18	.05
1.2	1.00	.98	.94	.87	.77	.65	.51	.35	.20	.09
1.3	1.00	.99	.94	.87	.77	.65	.51	.36	.22	.13
1.4	1.00	.99	.94	87	.77	.66	.52	.38	.24	.17
1.5	1.00	.99	.94	.87	.78	.66	.53	.39	.26	.20
1.6	1.00	.99	.94	.87	.78	.67	.54	.40	.29	.23
1.7	1.00	.99	.94	.88	.78	.67	.55	.42	.31	.26
1.8	1.00	.99	.94	.88	.79	.68	.56	.43	.33	.29
1.9	1.00	.99	.95	.88	.79	.69	.57	.45	.35	.31
2.0	1.00	.99	.95	.88	.80	.69	.58	.46	.37	.33
r / ø	180° 360	170° 350	160° 340	150° 330	140° 320	130° 310	120° 300	110° 290	100° 280	90° 270

Table 7.- Values of x in Formula (61)

ϕ / r	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
0.0	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
0.1	0.0	8.2	16.6	25.3	34.5	44.3	54.8	66.0	77.8	90.0
0.2	0.0	6.7	13.6	21.1	29.2	38.5	49.1	61.4	75.2	90.0
0.3	0.0	5.4	11.1	17.3	24.3	32.7	43.0	55.9	71.9	90.0
0.4	0.0	4.3	8.9	13.9	19.8	27.1	36.6	49.7	67.6	90.0
0.5	0.0	3.4	6.9	10.9	15.6	21.7	30.0	42.5	62.1	90.0
0.6	0.0	2.5	5.2	8.2	11.8	16.6	23.4	34.5	54.8	90.0
0.7	0.0	1.8	3.7	5.8	8.4	11.9	17.0	25.9	45.0	90.0
0.8	0.0	1.1	2.3	3.7	5.3	7.5	10.9	17.0	32.2	90.0
0.9	0.0	0.5	1.1	1.7	2.5	3.6	5.2	8.2	16.6	90.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	----
1.1	0.0	-0.5	-1.0	-1.6	-2.3	-3.2	-4.7	-7.4	-15.1	-90.0
1.2	0.0	-0.9	-1.9	-3.0	-4.4	-6.2	-8.9	-14.0	-27.3	-90.0
1.3	0.0	-1.3	-2.7	-4.3	-6.2	-8.8	-12.7	-19.7	-36.5	-90.0
1.4	0.0	-1.7	-3.5	-5.5	-8.0	-11.2	-16.1	-24.6	-43.4	-90.0
1.5	0.0	-2.0	-4.2	-6.6	-9.5	-13.4	-19.1	-28.8	-48.6	-90.0
1.6	0.0	-2.3	-4.8	-7.6	-11.0	-15.4	-21.8	-32.4	-52.6	-90.0
1.7	0.0	-2.6	-5.4	-8.5	-12.3	-17.2	-24.2	-35.5	-55.8	-90.0
1.8	0.0	-2.9	-5.9	-9.4	-13.5	-18.8	-26.3	-38.1	-58.3	-90.0
1.9	0.0	-3.1	-6.4	-10.2	-14.6	-20.3	-28.3	-40.4	-60.4	-90.0
2.0	0.0	-3.4	-6.9	-10.9	-15.6	-21.7	-30.0	-42.5	-62.1	-90.0
r / ϕ	180° 360	170° 350	160° 340	150° 330	140° 320	130° 310	120° 300	110° 290	100° 280	90° 270

When bottom argument is used, reverse sign of tabular value.

Table 8

Acceleration in time of HHW and LLW in degrees of diurnal wave (1)

Phase difference = MKO–½v for HHW and MKO±90°–½w for LLW

Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
Ratio of amplitude of diurnal wave to that of semidiurnal wave	° :	° :	° :	° :	° :	° :	° :	° :	° :	° :
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.2	0.5	0.7	0.9	1.1	1.2	1.3	1.4	1.4
0.2	0.0	0.5	0.9	1.4	1.8	2.1	2.4	2.6	2.8	2.9
0.3	0.0	0.7	1.4	2.0	2.6	3.1	3.6	3.9	4.2	4.3
0.4	0.0	0.9	1.8	2.6	3.4	4.1	4.7	5.2	5.6	5.7
0.5	0.0	1.1	2.2	3.2	4.2	5.1	5.8	6.5	6.9	7.2
0.6	0.0	1.3	2.6	3.8	5.0	6.0	6.9	7.7	8.3	8.6
0.7	0.0	1.5	2.9	4.4	5.7	6.9	8.0	8.9	9.6	10.1
0.8	0.0	1.7	3.3	4.9	6.4	7.8	9.0	10.1	11.0	11.5
0.9	0.0	1.8	3.6	5.4	7.1	8.7	10.1	11.3	12.3	13.0
1.0	0.0	2.0	4.0	5.9	7.7	9.5	11.1	12.5	13.6	14.5
1.1	0.0	2.2	4.3	6.4	8.4	10.3	12.0	13.6	15.0	16.0
1.2	0.0	2.3	4.6	6.8	9.0	11.1	13.0	14.8	16.3	17.5
1.3	0.0	2.5	4.9	7.3	9.6	11.8	14.0	15.9	17.6	19.0
1.4	0.0	2.6	5.2	7.7	10.2	12.6	14.9	17.0	18.9	20.5
1.5	0.0	2.7	5.4	8.1	10.8	13.3	15.8	18.1	20.2	22.0
1.6	0.0	2.9	5.7	8.5	11.3	14.0	16.6	19.2	21.5	23.6
1.7	0.0	3.0	5.9	8.9	11.8	14.7	17.5	20.2	22.8	25.2
1.8	0.0	3.1	6.2	9.3	12.3	15.4	18.4	21.3	24.1	26.8
1.9	0.0	3.2	6.4	9.6	12.8	16.0	19.2	22.3	25.4	28.4
2.0	0.0	3.3	6.7	10.0	13.3	16.7	20.0	23.3	26.7	30.0
2.1	0.0	3.4	6.9	10.3	13.8	17.3	20.8	24.3	28.0	31.7
2.2	0.0	3.5	7.1	10.7	14.3	17.9	21.6	25.3	29.2	33.4
2.3	0.0	3.6	7.3	11.0	14.7	18.5	22.3	26.3	30.5	35.1
2.4	0.0	3.7	7.5	11.3	15.1	19.0	23.1	27.3	31.8	36.9
2.5	0.0	3.8	7.7	11.6	15.6	19.6	23.8	28.2	33.0	38.7
2.6	0.0	3.9	7.9	11.9	16.0	20.2	24.5	29.1	34.3	40.5
2.7	0.0	4.0	8.1	12.2	16.4	20.7	25.2	30.1	35.5	42.4
2.8	0.0	4.1	8.3	12.5	16.8	21.2	25.9	31.0	36.8	44.4
2.9	0.0	4.2	8.4	12.7	17.1	21.7	26.5	31.8	38.0	46.5
3.0	0.0	4.3	8.6	13.0	17.5	22.2	27.2	32.7	39.2	48.6
Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :

Tabular values positive with top arguments, negative with bottom arguments. When phase difference is 90° or 270°, the corresponding diurnal inequality is zero and tabular values may be either positive or negative according to the tide selected for the HHW or the LLW.

Acceleration in time of HHW and LLW in degrees of diurnal wave (2)

Phase difference = MKO–½v for HHW and MKO±90°–½w for LLW

Phase Difference (Ratio of amplitude of diurnal wave to that of semidiurnal wave)	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
3.0	0.0	4.3	8.6	13.0	17.5	22.2	27.2	32.7	39.2	48.6
3.1	0.0	4.4	8.8	13.3	17.9	22.7	27.8	33.5	40.4	50.8
3.2	0.0	4.4	8.9	13.5	18.2	23.1	28.4	34.4	41.6	53.1
3.3	0.0	4.5	9.1	13.8	18.6	23.6	29.0	35.2	42.8	55.6
3.4	0.0	4.6	9.2	14.0	18.9	24.0	29.6	36.0	44.0	58.2
3.5	0.0	4.7	9.4	14.2	19.2	24.5	30.2	36.8	45.2	61.0
3.6	0.0	4.7	9.5	14.4	19.5	24.9	30.8	37.5	46.3	64.2
3.7	0.0	4.8	9.7	14.7	19.8	25.3	31.3	38.3	47.4	67.7
3.8	0.0	4.9	9.8	14.9	20.1	25.7	31.9	39.0	48.5	71.8
3.9	0.0	4.9	9.9	15.1	20.4	26.1	32.4	39.7	49.6	77.2
4.0	0.0	5.0	10.1	15.3	20.7	26.5	32.9	40.4	50.6	90.0
4.1	0.0	5.1	10.2	15.4	21.0	26.8	33.4	41.1	51.7	90.0
4.2	0.0	5.1	10.3	15.6	21.2	27.2	33.9	41.8	52.7	90.0
4.3	0.0	5.2	10.4	15.8	21.5	27.6	34.3	42.4	53.6	90.0
4.4	0.0	5.2	10.6	16.0	21.7	27.9	34.8	43.0	54.6	90.0
4.5	0.0	5.3	10.7	16.2	22.0	28.2	35.2	43.6	55.5	90.0
4.6	0.0	5.4	10.8	16.4	22.2	28.6	35.7	44.2	56.3	90.0
4.7	0.0	5.4	10.9	16.5	22.5	28.9	36.1	44.8	57.2	90.0
4.8	0.0	5.5	11.0	16.7	22.7	29.2	36.5	45.4	58.0	90.0
4.9	0.0	5.5	11.1	16.9	22.9	29.5	36.9	45.9	58.8	90.0
5.0	0.0	5.6	11.2	17.0	23.2	29.8	37.3	46.5	59.5	90.0
5.1	0.0	5.6	11.3	17.2	23.4	30.1	37.7	47.0	60.2	90.0
5.2	0.0	5.7	11.4	17.3	23.6	30.4	38.1	47.5	60.9	90.0
5.3	0.0	5.7	11.5	17.5	23.8	30.7	38.4	48.0	61.6	90.0
5.4	0.0	5.8	11.6	17.6	24.0	30.9	38.8	48.4	62.2	90.0
5.5	0.0	5.8	11.7	17.8	24.2	31.2	39.1	48.9	62.8	90.0
5.6	0.0	5.8	11.8	17.9	24.4	31.4	39.5	49.3	63.4	90.0
5.7	0.0	5.9	11.9	18.1	24.6	31.7	39.8	49.7	63.9	90.0
5.8	0.0	5.9	12.0	18.2	24.8	32.0	40.1	50.2	64.4	90.0
5.9	0.0	6.0	12.0	18.3	25.0	32.2	40.4	50.6	64.9	90.0
6.0	0.0	6.0	12.1	18.4	25.1	32.4	40.7	51.0	65.4	90.0
Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :

Tabular values positive with top arguments, negative with bottom arguments. When phase difference is 90° or 270°, the corresponding diurnal inequality is zero and tabular values may be either positive or negative according to the tide selected for the HHW or the LLW.

Acceleration in time of HHW and LLW in degrees of diurnal wave (3)

Phase difference = MKO-½v for HHW and MKO±90°-½w for LLW

Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
	°	°	°	°	°	°	°	°	°	°
Ratio of diurnal to semidiurnal wave amplitude 6	0.0	6.0	12.1	18.4	25.1	32.4	40.7	51.0	65.4	90.0
7	0.0	6.4	12.9	19.6	26.7	34.5	43.4	54.3	69.0	90.0
8	0.0	6.7	13.5	20.5	28.0	36.2	45.5	56.7	71.2	90.0
9	0.0	6.9	14.0	21.3	29.1	37.6	47.2	58.6	72.8	90.0
10	0.0	7.2	14.4	22.0	30.0	38.7	48.6	60.0	73.9	90.0
11	0.0	7.4	14.8	22.6	30.8	39.7	49.6	61.2	74.7	90.0
12	0.0	7.5	15.1	23.1	31.5	40.5	50.6	62.1	75.3	90.0
13	0.0	7.7	15.4	23.5	32.0	41.2	51.4	62.8	75.8	90.0
14	0.0	7.8	15.7	23.9	32.5	41.8	52.0	63.4	76.2	90.0
15	0.0	7.9	16.0	24.3	33.0	42.3	52.6	64.0	76.5	90.0
16	0.0	8.0	16.2	24.6	33.4	42.8	53.1	64.4	76.8	90.0
17	0.0	8.1	16.4	24.9	33.8	43.2	53.5	64.8	77.0	90.0
18	0.0	8.2	16.5	25.1	34.1	43.6	53.9	65.1	77.2	90.0
19	0.0	8.3	16.7	25.3	34.4	44.0	54.3	65.4	77.4	90.0
20	0.0	8.4	16.8	25.5	34.6	44.3	54.6	65.7	77.6	90.0
30	0.0	8.8	17.8	26.9	36.3	46.2	56.4	67.2	78.5	90.0
40	0.0	9.1	18.3	27.6	37.2	47.1	57.3	68.0	78.9	90.0
50	0.0	9.3	18.6	28.1	37.8	47.7	57.9	68.4	79.2	90.0
100	0.0	9.6	19.3	29.0	38.9	48.9	59.0	69.2	79.6	90.0
500	0.0	9.9	19.8	29.8	39.8	49.8	59.8	69.8	79.9	90.0
Infinite	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :

Tabular values positive with top arguments, negative with bottom arguments. When phase difference is 90° or 270°, the corresponding diurnal inequality is zero and tabular values may be either positive or negative according to the tide selected for the HHW or the LLW.

Acceleration in Lower High Water and Higher Low Water
Expressed in degrees of diurnal wave

Phase Difference	180°: 360	170°: 350	160°: 340	150°: 330	140°: 320	130°: 310	120°: 300	110°: 290	100°: 280	90°: 270:
Ratio of amplitude of diurnal wave to that of semidiurnal wave	°	°	°	°	°	°	°	°	°	°
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.3	0.5	0.7	0.9	1.1	1.3	1.4	1.4	1.4
0.2	0.0	0.5	1.0	1.5	1.9	2.3	2.5	2.7	2.8	2.9
0.3	0.0	0.8	1.6	2.3	2.9	3.5	3.9	4.2	4.3	4.3
0.4	0.0	1.1	2.2	3.1	4.0	4.7	5.2	5.6	5.8	5.7
0.5	0.0	1.4	2.8	4.0	5.1	6.0	6.6	7.0	7.2	7.2
0.6	0.0	1.8	3.4	5.0	6.3	7.3	8.1	8.5	8.7	8.6
0.7	0.0	2.1	4.1	5.9	7.5	8.7	9.6	10.1	10.2	10.1
0.8	0.0	2.5	4.8	7.0	8.8	10.2	11.1	11.7	11.8	11.5
0.9	0.0	2.9	5.6	8.0	10.1	11.7	12.7	13.3	13.4	13.0
1.0	0.0	3.3	6.4	9.2	11.5	13.3	14.4	14.9	14.9	14.5
1.1	0.0	3.8	7.3	10.4	13.0	14.9	16.1	16.6	16.6	16.0
1.2	0.0	4.2	8.2	11.8	14.6	16.7	18.0	18.4	18.2	17.5
1.3	0.0	4.8	9.3	13.2	16.4	18.6	19.9	20.3	19.9	19.0
1.4	0.0	5.3	10.4	14.8	18.3	20.7	21.9	22.2	21.6	20.5
1.5	0.0	5.9	11.6	16.5	20.3	22.9	24.1	24.2	23.4	22.0
1.6	0.0	6.6	12.9	18.4	22.6	25.4	26.5	26.3	25.3	23.6
1.7	0.0	7.3	14.3	20.5	25.3	28.1	29.1	28.6	27.2	25.2
1.8	0.0	8.1	15.9	23.0	28.4	31.4	32.0	31.0	29.1	26.8
1.9	0.0	9.0	17.8	26.0	32.5	35.7	35.5	33.7	31.2	28.4
2.0	0.0	10.0	20.0	30.0	40.0	43.3	40.0	36.7	33.3	30.0
2.1	0.0	11.1	22.7					40.2	35.6	31.7
2.2	0.0	12.4	26.4					44.6	38.1	33.4
2.3	0.0	13.9							40.7	35.1
2.4	0.0	15.6							43.6	36.9
2.5	0.0	17.9							47.1	38.7
2.6	0.0	21.1							51.4	40.5
2.7	0.0									42.4
2.8	0.0									44.4
2.9	0.0									46.5
3.0	0.0									48.6
Phase Difference	0°: 180	10°: 190	20°: 200	30°: 210	40°: 220	50°: 230	60°: 240	70°: 250	80°: 260	90°: 270:

Tide becomes diurnal with no lower high or higher low water.

Tabular values directly applicable for high water. Change phase difference by ± 90° for low water. Values positive with top arguments, negative with bottom arguments.

Table 9

Acceleration in time of HHW and LLW in solar hours (1)

Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90° 270 :
Ratio of amplitude of diurnal wave to that of semidiurnal wave	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	.00	.02	.03	.05	.06	.07	.08	.09	.10	.10
0.2	.00	.03	.06	.09	.12	.15	.17	.18	.19	.20
0.3	.00	.05	.09	.14	.18	.22	.25	.27	.29	.30
0.4	.00	.06	.12	.18	.24	.28	.33	.36	.38	.40
0.5	.00	.08	.15	.22	.29	.35	.40	.45	.48	.50
0.6	.00	.09	.18	.26	.34	.42	.48	.53	.57	.60
0.7	.00	.10	.20	.30	.39	.48	.55	.62	.66	.70
0.8	.00	.11	.23	.34	.44	.54	.62	.70	.76	.80
0.9	.00	.13	.25	.37	.49	.60	.69	.78	.85	.90
1.0	.00	.14	.27	.41	.53	.65	.76	.86	.94	1.00
1.1	.00	.15	.30	.44	.58	.71	.83	.94	1.03	1.10
1.2	.00	.16	.32	.47	.62	.76	.90	1.02	1.12	1.20
1.3	.00	.17	.34	.50	.66	.82	.96	1.10	1.21	1.31
1.4	.00	.18	.36	.53	.70	.87	1.03	1.17	1.30	1.41
1.5	.00	.19	.37	.56	.74	.92	1.09	1.25	1.39	1.52
1.6	.00	.20	.39	.59	.78	.97	1.15	1.32	1.48	1.63
1.7	.00	.21	.41	.61	.82	1.01	1.21	1.40	1.57	1.74
1.8	.00	.21	.43	.64	.85	1.06	1.27	1.47	1.66	1.85
1.9	.00	.22	.44	.67	.89	1.11	1.32	1.54	1.75	1.96
2.0	.00	.23	.46	.69	.92	1.15	1.38	1.61	1.84	2.07
2.1	.00	.24	.48	.71	.95	1.19	1.43	1.68	1.93	2.18
2.2	.00	.24	.49	.74	.98	1.23	1.49	1.75	2.02	2.30
2.3	.00	.25	.50	.76	1.01	1.27	1.54	1.81	2.10	2.42
2.4	.00	.26	.52	.78	1.04	1.31	1.59	1.88	2.19	2.54
2.5	.00	.27	.53	.80	1.07	1.35	1.64	1.95	2.28	2.67
2.6	.00	.27	.55	.82	1.10	1.39	1.69	2.01	2.37	2.80
2.7	.00	.28	.56	.84	1.13	1.43	1.74	2.07	2.45	2.93
2.8	.00	.28	.57	.86	1.16	1.46	1.79	2.14	2.54	3.07
2.9	.00	.29	.58	.88	1.18	1.50	1.83	2.20	2.62	3.21
3.0	.00	.30	.59	.90	1.21	1.53	1.88	2.26	2.71	3.35
Phase Difference	180 : 360 :	170 : 350 :	160 : 340 :	150 : 330 :	140 : 320 :	130 : 310 :	120 : 300 :	110 : 290 :	100 : 280 :	90 : 270 :

Tabular values positive with top arguments, negative with bottom arguments. When phase difference is 90° or 270°, the corresponding diurnal inequality is zero and tabular values may be either positive or negative according to the tide selected for the HHW or the LLW.

Table 9

Acceleration in time of HHW and LLW in solar hours (2)

Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
Ratio of amplitude of diurnal wave to that of semidiurnal wave	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:
3.0	0.00	0.30	0.59	0.90	1.21	1.53	1.88	2.26	2.71	3.35
3.1	.00	.30	.61	.91	1.23	1.56	1.92	2.31	2.79	3.50
3.2	.00	.31	.62	.93	1.26	1.60	1.96	2.37	2.87	3.67
3.3	.00	.31	.63	.95	1.28	1.63	2.00	2.43	2.95	3.84
3.4	.00	.32	.64	.96	1.30	1.66	2.04	2.48	3.04	4.02
3.5	.00	.32	.65	.98	1.33	1.69	2.08	2.54	3.12	4.21
3.6	.00	.33	.66	1.00	1.35	1.72	2.12	2.59	3.19	4.43
3.7	.00	.33	.67	1.01	1.37	1.75	2.16	2.64	3.27	4.67
3.8	.00	.34	.68	1.03	1.39	1.77	2.20	2.69	3.35	4.95
3.9	.00	.34	.69	1.04	1.41	1.80	2.23	2.74	3.42	5.32
4.0	.00	.35	.70	1.05	1.43	1.83	2.27	2.79	3.49	6.21
4.1	.00	.35	.70	1.07	1.45	1.85	2.30	2.84	3.56	6.21
4.2	.00	.35	.71	1.08	1.46	1.88	2.34	2.88	3.63	6.21
4.3	.00	.36	.72	1.09	1.48	1.90	2.37	2.93	3.70	6.21
4.4	.00	.36	.73	1.11	1.50	1.93	2.40	2.97	3.76	6.21
4.5	.00	.37	.74	1.12	1.52	1.95	2.43	3.01	3.83	6.21
4.6	.00	.37	.74	1.13	1.53	1.97	2.46	3.05	3.89	6.21
4.7	.00	.37	.75	1.14	1.55	1.99	2.49	3.09	3.95	6.21
4.8	.00	.38	.76	1.15	1.57	2.01	2.52	3.13	4.00	6.21
4.9	.00	.38	.77	1.16	1.58	2.04	2.55	3.17	4.06	6.21
5.0	.00	.38	.77	1.18	1.60	2.06	2.57	3.21	4.11	6.21
5.1	.00	.39	.78	1.19	1.61	2.08	2.60	3.24	4.16	6.21
5.2	.00	.39	.79	1.20	1.63	2.10	2.63	3.27	4.20	6.21
5.3	.00	.39	.79	1.21	1.64	2.12	2.65	3.31	4.25	6.21
5.4	.00	.40	.80	1.22	1.66	2.13	2.68	3.34	4.29	6.21
5.5	.00	.40	.81	1.23	1.67	2.15	2.70	3.37	4.33	6.21
5.6	.00	.40	.81	1.24	1.68	2.17	2.72	3.40	4.37	6.21
5.7	.00	.41	.82	1.25	1.70	2.19	2.75	3.43	4.41	6.21
5.8	.00	.41	.83	1.25	1.71	2.21	2.77	3.46	4.44	6.21
5.9	.00	.41	.83	1.26	1.72	2.22	2.79	3.49	4.48	6.21
6.0	.00	.42	.84	1.27	1.73	2.24	2.81	3.52	4.51	6.21
Phase Difference	180 : 360 :	170 : 350 :	160 : 340 :	150 : 330 :	140 : 320 :	130 : 310 :	120 : 300 :	110 : 290 :	100 : 280 :	90 : 270 :

Tabular values positive with toparguments, negative with bottom arguments. When phase difference is 90° or 270°, the corresponding diurnal inequality is zero and tabular values may be either positive or negative according to the tide selected for the HHW or the LLW.

Table 9

Acceleration in time of HHW and LLW in solar hours (3)

Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
Ratio of diurnal to semidiurnal wave amplitude	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:
6	0.00	0.42	0.84	1.27	1.73	2.24	2.81	3.52	4.51	6.21
7	.00	.44	.89	1.35	1.84	2.38	3.00	3.75	4.76	6.21
8	.00	.46	.93	1.42	1.93	2.50	3.14	3.92	4.92	6.21
9	.00	.48	.97	1.47	2.01	2.59	3.26	4.04	5.02	6.21
10	.00	.50	1.00	1.52	2.07	2.67	3.35	4.14	5.10	6.21
11	.00	.51	1.02	1.56	2.12	2.74	3.43	4.22	5.15	6.21
12	.00	.52	1.04	1.59	2.17	2.80	3.49	4.28	5.20	6.21
13	.00	.53	1.06	1.62	2.21	2.85	3.54	4.33	5.23	6.21
14	.00	.54	1.08	1.65	2.25	2.89	3.59	4.37	5.26	6.21
15	.00	.55	1.10	1.67	2.28	2.92	3.63	4.41	5.28	6.21
16	.00	.55	1.12	1.69	2.31	2.95	3.66	4.44	5.30	6.21
17	.00	.56	1.13	1.71	2.33	2.98	3.69	4.47	5.32	6.21
18	.00	.57	1.14	1.73	2.35	3.01	3.72	4.49	5.33	6.21
19	.00	.57	1.15	1.75	2.37	3.03	3.74	4.51	5.34	6.21
20	.00	.58	1.16	1.76	2.38	3.05	3.76	4.53	5.35	6.21
30	.00	.61	1.23	1.86	2.51	3.19	3.89	4.64	5.42	6.21
40	.00	.63	1.26	1.91	2.57	3.25	3.96	4.69	5.45	6.21
50	.00	.64	1.28	1.94	2.61	3.29	4.00	4.72	5.46	6.21
100	.00	.66	1.33	2.00	2.68	3.37	4.07	4.78	5.49	6.21
500	.00	.68	1.37	2.06	2.74	3.43	4.13	4.82	5.51	6.21
Infinite	0.00	0.69	1.38	2.07	2.76	3.45	4.14	4.83	5.52	6.21
Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :

Tabular values positive with top arguments, negative with bottom arguments. When phase difference is 90° or 270°, the corresponding diurnal inequality is zero and tabular values may be either positive or negative according to the tide selected for the HHW or the LLW.

Acceleration in Lower High Water and Higher Low Water
Expressed in solar hours

Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :
	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:	hour:
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	.00	.02	.03	.05	.06	.08	.09	.09	.10	.10
0.2	.00	.04	.07	.10	.13	.16	.18	.19	.20	.20
0.3	.00	.06	.11	.16	.20	.24	.27	.29	.30	.30
0.4	.00	.08	.15	.22	.28	.32	.36	.39	.40	.40
0.5	.00	.10	.19	.28	.35	.41	.46	.49	.50	.50
0.6	.00	.12	.24	.34	.43	.50	.56	.59	.60	.60
0.7	.00	.15	.28	.41	.52	.60	.66	.70	.71	.70
0.8	.00	.17	.33	.48	.60	.70	.77	.80	.81	.80
0.9	.00	.20	.39	.56	.70	.81	.88	.92	.92	.90
1.0	.00	.23	.44	.64	.80	.91	.99	1.03	1.03	1.00
1.1	.00	.26	.50	.72	.90	1.03	1.11	1.15	1.14	1.10
1.2	.00	.29	.57	.81	1.01	1.15	1.24	1.27	1.26	1.20
1.3	.00	.33	.64	.91	1.13	1.29	1.37	1.40	1.37	1.31
1.4	.00	.37	.71	1.02	1.26	1.43	1.51	1.53	1.49	1.41
1.5	.00	.41	.80	1.14	1.40	1.58	1.66	1.67	1.62	1.52
1.6	.00	.46	.89	1.27	1.56	1.75	1.83	1.82	1.74	1.63
1.7	.00	.51	.99	1.41	1.74	1.94	2.01	1.97	1.87	1.74
1.8	.00	.56	1.10	1.58	1.96	2.17	2.21	2.14	2.01	1.85
1.9	.00	.62	1.23	1.79	2.24	2.46	2.45	2.32	2.15	1.96
2.0	.00	.69	1.38	2.07	2.76	2.99	2.76	2.53	2.30	2.07
2.1	.00	.77	1.57					2.77	2.46	2.18
2.2	.00	.85	1.82					3.08	2.63	2.30
2.3	.00	.96							2.81	2.42
2.4	.00	1.08		Tide becomes diurnal with					3.01	2.54
2.5	.00	1.23		no lower high or higher low					3.25	2.67
2.6	.00	1.45		water.					3.55	2.80
2.7	.00									2.93
2.8	.00									3.07
2.9	.00									3.21
3.0	.00									3.35
Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :

Tabular values directly applicable for high water. Change phase difference by ± 90° for low water. Values positive with top arguments, negative with bottom arguments.

Table 10

Height factors for HHW and LLW (1)

Phase Difference	0°: 180	10°: 190	20°: 200	30°: 210	40°: 220	50°: 230	60°: 240	70°: 250	80°: 260	90° 270
Ratio of amplitude of diurnal wave to that of semidiurnal wave										
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.100	1.098	1.094	1.087	1.077	1.065	1.051	1.035	1.019	1.001
0.2	1.200	1.197	1.188	1.174	1.155	1.131	1.104	1.073	1.040	1.005
0.3	1.300	1.296	1.283	1.262	1.234	1.199	1.158	1.112	1.063	1.011
0.4	1.400	1.394	1.378	1.351	1.314	1.268	1.214	1.154	1.088	1.020
0.5	1.500	1.493	1.473	1.440	1.395	1.338	1.272	1.198	1.116	1.031
0.6	1.600	1.592	1.568	1.530	1.476	1.410	1.331	1.243	1.147	1.045
0.7	1.700	1.691	1.664	1.620	1.558	1.482	1.392	1.290	1.179	1.061
0.8	1.800	1.790	1.760	1.710	1.642	1.556	1.454	1.340	1.214	1.080
0.9	1.900	1.889	1.855	1.800	1.725	1.630	1.518	1.391	1.251	1.101
1.0	2.000	1.988	1.952	1.892	1.809	1.706	1.583	1.444	1.290	1.125
1.1	2.100	2.087	2.048	1.983	1.894	1.782	1.650	1.498	1.331	1.151
1.2	2.200	2.186	2.144	2.075	1.980	1.860	1.717	1.554	1.374	1.180
1.3	2.300	2.285	2.240	2.167	2.066	1.938	1.786	1.612	1.419	1.211
1.4	2.400	2.384	2.337	2.259	2.152	2.017	1.856	1.672	1.467	1.245
1.5	2.500	2.483	2.434	2.352	2.239	2.097	1.927	1.732	1.516	1.281
1.6	2.600	2.583	2.531	2.445	2.327	2.177	1.999	1.795	1.567	1.320
1.7	2.700	2.681	2.628	2.538	2.414	2.259	2.072	1.859	1.620	1.361
1.8	2.800	2.781	2.725	2.632	2.503	2.341	2.147	1.924	1.676	1.405
1.9	2.900	2.880	2.822	2.725	2.592	2.423	2.222	1.991	1.733	1.451
2.0	3.000	2.980	2.919	2.819	2.681	2.506	2.298	2.059	1.792	1.500
2.1	3.100	3.079	3.016	2.913	2.770	2.590	2.375	2.128	1.852	1.551
2.2	3.200	3.178	3.114	3.007	2.860	2.675	2.453	2.198	1.914	1.605
2.3	3.300	3.278	3.211	3.102	2.951	2.760	2.532	2.270	1.978	1.661
2.4	3.400	3.377	3.309	3.196	3.041	2.845	2.611	2.343	2.044	1.720
2.5	3.500	3.477	3.407	3.291	3.132	2.931	2.692	2.417	2.112	1.781
2.6	3.600	3.576	3.504	3.386	3.223	3.018	2.773	2.492	2.181	1.845
2.7	3.700	3.676	3.602	3.481	3.315	3.105	2.854	2.568	2.251	1.911
2.8	3.800	3.775	3.700	3.577	3.406	3.192	2.937	2.646	2.323	1.980
2.9	3.900	3.874	3.798	3.672	3.498	3.280	3.020	2.724	2.397	2.051
3.0	4.000	3.974	3.896	3.768	3.591	3.368	3.104	2.803	2.472	2.125
Phase Difference	180°: 360	170°: 350	160°: 340	150°: 330	140°: 320	130°: 310	120°: 300	110°: 290	100°: 280	90° 270

Factor multiplied by amplitude of semidiurnal wave gives height of HHW above MSL or depression of LLW below MSL.

Table 10

Height factors for HHW and LLW (2)

Phase Difference	0°: 180	10°: 190	20°: 200	30°: 210	40°: 220	50°: 230	60°: 240	70°: 250	80°: 260	90° 270
Ratio of amplitude of diurnal wave to that of semidiurnal wave.										
3.0	4.000	3.974	3.896	3.768	3.591	3.368	3.104	2.803	2.472	2.125
3.1	4.100	4.073	3.994	3.863	3.683	3.457	3.188	2.882	2.548	2.201
3.2	4.200	4.173	4.092	3.959	3.776	3.546	3.273	2.964	2.626	2.280
3.3	4.300	4.272	4.190	4.055	3.869	3.635	3.359	3.045	2.705	2.361
3.4	4.400	4.372	4.289	4.151	3.962	3.725	3.445	3.128	2.786	2.445
3.5	4.500	4.471	4.387	4.247	4.056	3.815	3.531	3.211	2.867	2.531
3.6	4.600	4.571	4.485	4.344	4.149	3.906	3.618	3.295	2.950	2.620
3.7	4.700	4.671	4.584	4.440	4.243	3.996	3.706	3.380	3.033	2.711
3.8	4.800	4.770	4.682	4.537	4.337	4.087	3.794	3.465	3.118	2.805
3.9	4.900	4.870	4.780	4.633	4.431	4.178	3.882	3.551	3.204	2.901
4.0	5.000	4.970	4.879	4.730	4.525	4.270	3.971	3.638	3.291	3.000
Phase Difference	180°: 360	170°: 350	160°: 340	150°: 330	140°: 320	130°: 310	120°: 300	110°: 290	100°: 280	90° 270

Factor multiplied by amplitude of semidiurnal wave gives height of HHW above MSL or depression of LLW below MSL.

If ratio of amplitude of diurnal wave to that of semidiurnal wave is greater than 4.0, use Table 16.

Table 10a

Height factors for LHW and HLW

Phase Difference	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90° 270
Ratio of amplitude of diurnal wave to that of semidiurnal wave										
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.900	0.902	0.906	0.914	0.924	0.936	0.951	0.967	0.984	1.001
0.2	.800	.803	.813	.828	.849	.874	.904	.936	.970	1.005
0.3	.700	.705	.720	.743	.775	.814	.859	.908	.959	1.011
0.4	.600	.607	.627	.659	.702	.755	.816	.882	.950	1.020
0.5	.500	.509	.534	.576	.631	.699	.775	.858	.944	1.031
0.6	.400	.411	.442	.493	.561	.644	.736	.837	.941	1.045
0.7	.300	.313	.351	.412	.493	.591	.700	.818	.940	1.061
0.8	.200	.215	.260	.331	.426	.540	.667	.802	.942	1.080
0.9	.100	.117	.169	.252	.361	.491	.636	.789	.946	1.101
1.0	.000	.020	.079	.174	.298	.445	.607	.779	.953	1.125
1.1	-.100	-.077	-.010	.097	.237	.401	.582	.772	.963	1.151
1.2	-.200	-.174	-.098	.021	.178	.360	.560	.767	.976	1.180
1.3	-.300	-.271	-.186	-.052	.121	.322	.540	.766	.992	1.211
1.4	-.400	-.367	-.273	-.124	.067	.287	.524	.768	1.010	1.245
1.5	-.500	-.464	-.358	-.194	.016	.256	.512	.774	1.032	1.281
1.6	-.600	-.560	-.443	-.262	-.033	.229	.504	.783	1.057	1.320
1.7	-.700	-.655	-.526	-.327	-.076	.206	.500	.796	1.085	1.361
1.8	-.800	-.751	-.608	-.389	-.115	.188	.501	.813	1.116	1.405
1.9	-.900	-.845	-.688	-.447	-.149	.176	.508	.834	1.150	1.451
2.0	-1.000	-.940	-.766	-.500	-.174	.174	.521	.860	1.188	1.500
2.1	-1.100	-1.033	-.841					.892	1.230	1.551
2.2	-1.200	-1.126	-.913					.930	1.275	1.605
2.3	-1.300	-1.218							1.324	1.661
2.4	-1.400	-1.309							1.377	1.720
2.5	-1.500	-1.398							1.435	1.781
2.6	-1.600	-1.486							1.498	1.845
2.7	-1.700									1.911
2.8	-1.800									1.980
2.9	-1.900									2.051
3.0	-2.000									2.125
Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90° 270

Tide becomes diurnal with no lower high or higher low water.

Tabular factor directly applicable for LHW. Change phase difference by ± 90° for HLW. Factor applied to amplitude of semidiurnal wave gives height of LHW above MSL or depression of HLW below MSL.

Tropic HHW and LLW factors with P_1 corrections

Phase Difference / Ratio of amplitude of diurnal wave to that of semidiurnal wave	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.100	1.098	1.094	1.087	1.077	1.065	1.051	1.035	1.019	1.013
0.2	1.200	1.197	1.188	1.174	1.155	1.131	1.104	1.073	1.041	1.029
0.3	1.300	1.296	1.283	1.262	1.234	1.199	1.158	1.112	1.064	1.048
0.4	1.400	1.394	1.378	1.351	1.314	1.268	1.214	1.154	1.090	1.069
0.5	1.500	1.493	1.473	1.440	1.395	1.338	1.272	1.198	1.118	1.092
0.6	1.600	1.592	1.568	1.530	1.476	1.410	1.331	1.243	1.150	1.119
0.7	1.700	1.691	1.664	1.620	1.558	1.482	1.392	1.290	1.182	1.147
0.8	1.800	1.790	1.760	1.710	1.642	1.556	1.454	1.340	1.218	1.178
0.9	1.900	1.889	1.855	1.800	1.725	1.630	1.518	1.391	1.255	1.211
1.0	2.000	1.988	1.952	1.892	1.809	1.706	1.583	1.444	1.295	1.248
1.1	2.100	2.087	2.048	1.983	1.894	1.782	1.650	1.498	1.337	1.286
1.2	2.200	2.186	2.144	2.075	1.980	1.860	1.717	1.554	1.381	1.327
1.3	2.300	2.285	2.240	2.167	2.066	1.938	1.786	1.612	1.428	1.371
1.4	2.400	2.384	2.337	2.259	2.152	2.017	1.856	1.672	1.477	1.417
1.5	2.500	2.483	2.434	2.352	2.239	2.097	1.927	1.732	1.528	1.465
1.6	2.600	2.583	2.531	2.445	2.327	2.177	1.999	1.795	1.581	1.517
1.7	2.700	2.681	2.628	2.538	2.414	2.259	2.072	1.859	1.635	1.570
1.8	2.800	2.781	2.725	2.632	2.503	2.341	2.147	1.924	1.694	1.626
1.9	2.900	2.880	2.822	2.725	2.592	2.423	2.222	1.991	1.754	1.684
2.0	3.000	2.980	2.919	2.819	2.681	2.506	2.298	2.059	1.816	1.746
2.1	3.100	3.079	3.016	2.913	2.770	2.590	2.375	2.128	1.880	1.809
2.2	3.200	3.178	3.114	3.007	2.860	2.675	2.453	2.198	1.945	1.875
2.3	3.300	3.278	3.211	3.102	2.951	2.760	2.532	2.270	2.014	1.944
2.4	3.400	3.377	3.309	3.196	3.041	2.845	2.611	2.343	2.086	2.015
2.5	3.500	3.477	3.407	3.291	3.132	2.931	2.692	2.417	2.160	2.088
2.6	3.600	3.576	3.504	3.386	3.223	3.018	2.773	2.492	2.236	2.165
2.7	3.700	3.676	3.602	3.481	3.315	3.105	2.854	2.568	2.310	2.243
2.8	3.800	3.775	3.700	3.577	3.406	3.192	2.937	2.646	2.390	2.324
2.9	3.900	3.874	3.798	3.672	3.498	3.280	3.020	2.724	2.470	2.407
3.0	4.000	3.974	3.896	3.768	3.591	3.368	3.104	2.803	2.550	2.494
Phase Difference	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :

Factor is to be multiplied by amplitude of semidiurnal wave.

Table 10t

Tropic HHW and LLW factors with P_1 corrections - cont'd

Phase Difference		0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90°: 270 :
Ratio of amplitude of diurnal wave to that of semidiurnal wave.	3.0	4.000	3.974	3.896	3.768	3.591	3.368	3.104	2.803	2.550	2.494
	3.1	4.100	4.073	3.994	3.863	3.683	3.457	3.188	2.882	2.640	2.582
	3.2	4.200	4.173	4.092	3.959	3.776	3.546	3.273	2.964	2.730	2.674
	3.3	4.300	4.272	4.190	4.055	3.869	3.635	3.359	3.045	2.820	2.767
	3.4	4.400	4.372	4.289	4.151	3.962	3.725	3.445	3.128	2.910	2.863
	3.5	4.500	4.471	4.387	4.247	4.056	3.815	3.531	3.211	3.010	2.962
	3.6	4.600	4.571	4.485	4.344	4.149	3.906	3.618	3.295	3.110	3.062
	3.7	4.700	4.671	4.584	4.440	4.243	3.996	3.706	3.380	3.210	3.166
	3.8	4.800	4.770	4.682	4.537	4.337	4.087	3.794	3.465	3.310	3.272
	3.9	4.900	4.870	4.780	4.633	4.431	4.178	3.882	3.551	3.410	3.380
	4.0	5.000	4.970	4.879	4.730	4.525	4.270	3.971	3.638	3.510	3.492
Phase Difference		180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90°: 270 :

Factor is to be multiplied by amplitude of semidiurnal wave.

Diurnal inequality factors

Ratio of amplitude of diurnal wave to that of semidiurnal wave

Phase Difference	0°: 180	10°: 170	20°: 160	30°: 150	40°: 140	50°: 130	60°: 120	70°: 110	80°: 100	90° 90
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.1	.200	.197	.188	.173	.153	.129	.100	.068	.035	.000
0.2	.400	.394	.375	.346	.306	.257	.200	.137	.070	.000
0.3	.600	.591	.563	.519	.459	.385	.299	.204	.104	.000
0.4	.800	.787	.751	.692	.612	.513	.398	.272	.138	.000
0.5	1.000	.984	.939	.864	.764	.640	.497	.340	.172	.000
0.6	1.200	1.181	1.126	1.037	.915	.766	.595	.406	.206	.000
0.7	1.400	1.378	1.313	1.208	1.065	.891	.692	.472	.239	.000
0.8	1.600	1.575	1.500	1.379	1.215	1.016	.787	.538	.272	.000
0.9	1.800	1.772	1.686	1.549	1.364	1.139	.882	.602	.305	.000
1.0	2.000	1.968	1.872	1.718	1.511	1.261	.976	.665	.337	.000
1.1	2.200	2.164	2.057	1.886	1.657	1.381	1.068	.726	.368	.000
1.2	2.400	2.360	2.242	2.054	1.802	1.500	1.158	.787	.398	.000
1.3	2.600	2.556	2.426	2.219	1.945	1.616	1.246	.846	.428	.000
1.4	2.800	2.752	2.610	2.383	2.085	1.730	1.332	.903	.457	.000
1.5	3.000	2.947	2.792	2.546	2.223	1.841	1.415	.958	.484	.000
1.6	3.200	3.142	2.974	2.707	2.359	1.948	1.495	1.012	.510	.000
1.7	3.400	3.337	3.154	2.865	2.490	2.053	1.572	1.063	.536	.000
1.8	3.600	3.532	3.333	3.021	2.618	2.153	1.646	1.111	.560	.000
1.9	3.800	3.726	3.510	3.172	2.741	2.247	1.714	1.157	.583	.000
2.0	4.000	3.920	3.685	3.319	2.855	2.332	1.777	1.199	.604	.000
2.1	4.200	4.112	3.857					1.236	.622	.000
2.2	4.400	4.304	4.027		Tide diurnal			1.266	.639	.000
2.3	4.600	4.496							.654	.000
2.4	4.800	4.686							.667	.000
2.5	5.000	4.875							.677	.000
2.6	5.200	5.062							.683	.000
2.7	5.400									.000
2.8	5.600									.000
2.9	5.800									.000
3.0	6.000									.000
Phase Difference	180°: 360	190°: 350	200°: 340	210°: 330	220°: 320	230°: 310	240°: 300	250°: 290	260°: 280	270° 270

For HHW-LHW enter table with Phase difference = P
For HLW-LLW enter table with Phase difference = P±90°
In either case multiply tabular factor by amplitude of semidiurnal wave.

 Table 11t. - Tropic inequality factors with P_1 corrections

Ratio of amplitude of diurnal wave to that of semidiurnal wave

Phase Difference	0° 180	10° 170	20° 160	30° 150	40° 140	50° 130	60° 120	70° 110	80° 100	90° 90
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.1	.200	.197	.188	.173	.153	.129	.100	.068	.036	.025
0.2	.400	.394	.375	.346	.306	.257	.200	.137	.071	.049
0.3	.600	.591	.563	.519	.459	.385	.299	.204	.106	.074
0.4	.800	.787	.751	.692	.612	.513	.398	.272	.141	.098
0.5	1.000	.984	.939	.864	.764	.640	.497	.340	.176	.123
0.6	1.200	1.181	1.126	1.037	.915	.766	.595	.406	.211	.148
0.7	1.400	1.378	1.313	1.208	1.065	.891	.692	.472	.246	.172
0.8	1.600	1.575	1.500	1.379	1.215	1.016	.787	.538	.280	.197
0.9	1.800	1.772	1.686	1.549	1.364	1.139	.882	.602	.314	.221
1.0	2.000	1.968	1.872	1.718	1.511	1.261	.976	.665	.347	.246
1.1	2.200	2.164	2.057	1.886	1.657	1.381	1.068	.726	.380	.271
1.2	2.400	2.360	2.242	2.054	1.802	1.500	1.158	.787	.412	.295
1.3	2.600	2.556	2.426	2.219	1.945	1.616	1.246	.846	.445	.320
1.4	2.800	2.752	2.610	2.383	2.085	1.730	1.332	.903	.477	.344
1.5	3.000	2.947	2.792	2.546	2.223	1.841	1.415	.958	.508	.369
1.6	3.200	3.142	2.974	2.707	2.359	1.948	1.495	1.012	.537	.394
1.7	3.400	3.337	3.154	2.865	2.490	2.053	1.572	1.063	.567	.418
1.8	3.600	3.532	3.333	3.021	2.618	2.153	1.646	1.111	.596	.443
1.9	3.800	3.726	3.510	3.172	2.741	2.247	1.714	1.157	.624	.467
2.0	4.000	3.920	3.685	3.319	2.855	2.332	1.777	1.199	.652	.492
2.1	4.200	4.112	3.857					1.236	.677	.517
2.2	4.400	4.304	4.027					1.266	.702	.541
2.3	4.600	4.496			Tide diurnal				.727	.566
2.4	4.800	4.686							.750	.590
2.5	5.000	4.875							.773	.615
2.6	5.200	5.062							.793	.640
2.7	5.400									.664
2.8	5.600									.689
2.9	5.800									.713
3.0	6.000									.738
Phase Difference	180° 360	190° 350	200° 340	210° 330	220° 320	230° 310	240° 300	250° 290	260° 280	270° 270

Factor is to be multiplied by amplitude of semidiurnal wave.

Table 12. - Effect of P_1 on diurnal inequality

L	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.637	0.627	0.617	0.607	0.598	0.588	0.578	0.569	0.559	0.550
0.1	.540	.530	.521	.511	.502	.493	.484	.475	.466	.458
0.2	.449	.440	.431	.423	.415	.406	.398	.390	.382	.373
0.3	.365	.357	.349	.341	.333	.326	.318	.310	.303	.296
0.4	.288	.281	.273	.266	.259	.252	.245	.238	.231	.224
0.5	.218	.211	.204	.198	.191	.185	.179	.173	.167	.161
0.6	.155	.149	.143	.138	.132	.126	.120	.115	.110	.105
0.7	.100	.095	.090	.085	.080	.075	.071	.067	.063	.058
0.8	.054	.050	.046	.043	.039	.035	.031	.028	.025	.022
0.9	.019	.016	.013	.011	.009	.007	.005	.003	.002	.001
1.0	.000	---	---	---	---	---	---	---	---	---

Table = $0.6366\ (1-L^2)^{1/2} - 0.0111\ L\ (\cos^{-1}L)^\circ$

$L = HWQ'/2P_1$ for HW inequality, or $LWQ'/2P_1$ for LW inequality.

Note:- Tabular value multiplied by amplitude of P_1 will give the change in height of HHW or LLW due to presence of this constituent. If above factor is to be combined with factors in Table 10, it should first be multiplied by ratio P_1/M_2.

Table 13. - Mean diurnal inequality factors
(Including P_1 effect)

O_1/K_1 \ P	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
0.0	0.646	0.646	0.646	0.646	0.646	0.646	0.646	0.646	0.646	0.646
0.1	.643	.642	.633	.619	.601	.582	.563	.547	.536	.531
0.2	.642	.640	.624	.598	.566	.531	.496	.465	.446	.437
0.3	.641	.638	.616	.585	.543	.495	.446	.402	.369	.357
0.4	.641	.635	.611	.574	.522	.467	.406	.349	.308	.291
0.5	.641	.634	.608	.565	.511	.446	.375	.309	.256	.235
0.6	.640	.632	.605	.560	.502	.432	.354	.278	.215	.189
0.7	.639	.631	.604	.557	.495	.422	.339	.255	.183	.151
0.8	.638	.630	.602	.555	.492	.416	.329	.239	.162	.127
0.9	.638	.629	.600	.554	.489	.413	.324	.231	.152	.113
1.0	.639	.629	.600	.552	.488	.410	.321	.227	.145	.106
O_1/K_1 / P	180° 360	170° 350	160° 340	150° 330	140° 320	130° 310	120° 300	110° 290	100° 280	90° 270

When O_1/K_1 is greater than unity, enter table with its reciprocal.

For HW inequality take P = MKO -½v; for LW inequality take P = MKO-½w±90°.

For mean inequalities multiply factor by (K_1+O_1).

Table 14.– Acceleration in diurnal tide due to semidiurnal wave.
(Expressed in degrees of diurnal wave)

R' \ P	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
	−	−	−	−	−	−	−	−	−	−
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01	0.0	0.4	0.7	1.0	1.1	1.1	1.0	0.8	0.4	0.0
.02	0.0	0.7	1.4	1.9	2.2	2.3	2.1	1.6	0.8	0.0
.03	0.0	1.1	2.0	2.8	3.3	3.4	3.1	2.4	1.3	0.0
.04	0.0	1.4	2.6	3.6	4.3	4.6	4.3	3.3	1.8	0.0
.05	0.0	1.6	3.2	4.5	5.4	5.7	5.4	4.3	2.4	0.0
.06	0.0	1.9	3.7	5.2	6.4	6.9	6.6	5.3	3.0	0.0
.07	0.0	2.2	4.2	6.0	7.3	8.0	7.8	6.4	3.7	0.0
.08	0.0	2.4	4.7	6.7	8.2	9.1	9.0	7.5	4.4	0.0
.09	0.0	2.6	5.1	7.4	9.1	10.2	10.2	8.7	5.2	0.0
.10	0.0	2.8	5.6	8.0	10.0	11.3	11.4	10.0	6.1	0.0
.11	0.0	3.0	5.9	8.6	10.8	12.3	12.7	11.3	7.1	0.0
.12	0.0	3.2	6.3	9.2	11.6	13.3	13.9	12.6	8.2	0.0
.13	0.0	3.4	6.7	9.7	12.3	14.3	15.1	13.9	9.4	0.0
.14	0.0	3.6	7.0	10.3	13.1	15.2	16.2	15.3	10.6	0.0
.15	0.0	3.7	7.4	10.8	13.8	16.1	17.4	16.7	12.1	0.0
.16	0.0	3.9	7.7	11.2	14.4	17.0	18.5	18.1	13.6	0.0
.17	0.0	4.0	8.0	11.7	15.1	17.8	19.6	19.5	15.2	0.0
.18	0.0	4.2	8.3	12.1	15.7	18.7	20.7	20.9	16.9	0.0
.19	0.0	4.3	8.5	12.6	16.3	19.4	21.7	22.2	18.6	0.0
.20	0.0	4.4	8.8	13.0	16.8	20.2	22.7	23.5	20.5	0.0
.21	0.0	4.6	9.0	13.3	17.4	20.9	23.6	24.8	22.3	0.0
.22	0.0	4.7	9.3	13.7	17.9	21.6	24.6	26.1	24.1	0.0
.23	0.0	4.8	9.5	14.1	18.4	22.3	25.5	27.3	25.9	0.0
.24	0.0	4.9	9.7	14.4	18.8	22.9	26.3	28.5	27.7	0.0
.25	0.0	5.0	9.9	14.7	19.3	23.5	27.1	29.6	29.4	0.0
.26	0.0	5.1	10.1	15.0	19.7	24.1	27.9	30.7	31.0	15.9
.27	0.0	5.2	10.3	15.3	20.2	24.7	28.7	31.7	32.5	22.2
.28	0.0	5.3	10.5	15.6	20.6	25.2	29.4	32.7	34.0	26.8
.29	0.0	5.4	10.7	15.9	21.0	25.7	30.1	33.6	35.4	30.5
.30	0.0	5.4	10.9	16.2	21.3	26.3	30.8	34.5	36.8	33.6
	+	+	+	+	+	+	+	+	+	+
R' / P	180° 360	170° 350	160° 340	150° 330	140° 320	130° 310	120° 300	110° 290	100° 280	90° 270

Tabular value is negative with top argument, positive with bottom argument.

Table directly applicable to HW; change argument P by ± 90° for LW.

Table 15

Acceleration in diurnal tide due to semidiurnal wave
(Expressed in solar hours)

R' \ P	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
	hour	hour	hour	hour	hour	hour	hour	hour	hour	hour
	–	–	–	–	–	–	–	–	–	–
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
.01	.00	.03	.05	.07	.08	.08	.07	.05	.03	.00
.02	.00	.05	.10	.13	.15	.16	.14	.11	.06	.00
.03	.00	.08	.14	.19	.23	.24	.22	.17	.09	.00
.04	.00	.10	.18	.25	.30	.32	.30	.23	.13	.00
.05	.00	.11	.22	.31	.37	.40	.38	.30	.17	.00
.06	.00	.13	.26	.36	.44	.48	.46	.37	.21	.00
.07	.00	.15	.29	.41	.50	.55	.54	.44	.26	.00
.08	.00	.17	.32	.46	.57	.63	.62	.52	.31	.00
.09	.00	.18	.35	.51	.63	.70	.70	.60	.36	.00
.10	.00	.19	.38	.55	.69	.78	.79	.69	.42	.00
.11	.00	.21	.41	.59	.75	.85	.88	.78	49	.00
.12	.00	.22	.43	.63	.80	.92	.96	.87	.57	.00
.13	.00	.23	.46	.67	.85	.99	1.04	.96	.65	.00
.14	.00	.25	.48	.71	.90	1.05	1.12	1.06	.74	.00
.15	.00	.26	.51	.75	.95	1.11	1.20	1.15	.83	.00
.16	.00	.27	.53	.78	.99	1.17	1.28	1.25	.94	.00
.17	.00	.28	.55	.81	1.04	1.23	1.35	1.35	1.05	.00
.18	.00	.29	.57	.84	1.08	1.29	1.43	1.44	1.17	.00
.19	.00	.30	.59	.87	1.12	1.34	1.50	1.53	1.29	.00
.20	.00	.31	.61	.90	1.16	1.39	1.57	1.62	1.41	.00
.21	.00	.32	.62	.92	1.20	1.44	1.63	1.71	1.54	.00
.22	.00	.32	.64	.95	1.24	1.49	1.70	1.80	1.66	.00
.23	.00	.33	.66	.97	1.27	1.54	1.76	1.88	1.79	.00
.24	.00	.34	.67	.99	1.30	1.58	1.81	1.96	1.91	.00
.25	.00	.34	.68	1.02	1.33	1.62	1.87	2.04	2.03	.00
.26	.00	.35	.70	1.04	1.36	1.66	1.93	2.12	2.14	1.10
.27	.00	.36	.71	1.06	1.39	1.70	1.98	2.19	2.24	1.53
.28	.00	.37	.72	1.08	1.42	1.74	2.03	2.26	2.35	1.85
.29	.00	.37	.74	1.10	1.45	1.77	2.08	2.32	2.44	2.10
.30	.00	.37	.75	1.12	1.47	1.81	2.13	2.38	2.54	2.32
		+	+	+	+	+	+	+	+	+
R' / P	180° 360	170° 350	160° 340	150° 330	140° 320	130° 310	120° 300	110° 290	100° 280	90° 270

Tabular value is negative with top argument, positive with bottom argument.
Table directly applicable to HW; change argument P by ± 90° for LW.

Table 16.-Height factors for diurnal tides.

R' \ P	0°: 180 :	10°: 190 :	20°: 200 :	30°: 210 :	40°: 220 :	50°: 230 :	60°: 240 :	70°: 250 :	80°: 260 :	90° 270
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.01	1.010	1.009	1.008	1.005	1.002	0.998	0.995	0.992	0.991	0.990
.02	1.020	1.019	1.016	1.011	1.004	.997	.991	.985	.981	.980
.03	1.030	1.028	1.024	1.016	1.007	.996	.986	.978	.972	.970
.04	1.040	1.038	1.032	1.022	1.010	.996	.983	.971	.963	.960
.05	1.050	1.048	1.040	1.028	1.013	.996	.979	.964	.954	.950
.06	1.060	1.057	1.048	1.035	1.017	.997	.976	.958	.945	.940
.07	1.070	1.067	1.057	1.041	1.021	.998	.973	.951	.936	.930
.08	1.080	1.076	1.065	1.048	1.025	.999	.971	.945	.927	.920
.09	1.090	1.086	1.074	1.055	1.030	1.000	.969	.940	.918	.910
.10	1.100	1.096	1.083	1.062	1.035	1.002	.968	.935	.910	.900
.11	1.110	1.105	1.092	1.069	1.040	1.005	.967	.930	.901	.890
.12	1.120	1.115	1.101	1.077	1.045	1.008	.966	.926	.893	.880
.13	1.130	1.125	1.109	1.084	1.051	1.011	.966	.922	.885	.870
.14	1.140	1.134	1.118	1.092	1.057	1.014	.966	.918	.878	.860
.15	1.150	1.144	1.127	1.100	1.063	1.017	.967	.915	.870	.850
.16	1.160	1.154	1.136	1.108	1.069	1.021	.968	.912	.863	.840
.17	1.170	1.164	1.146	1.116	1.075	1.025	.969	.910	.857	.830
.18	1.180	1.174	1.155	1.124	1.082	1.030	.971	.908	.851	.820
.19	1.190	1.184	1.164	1.132	1.088	1.035	.973	.907	.845	.810
.20	1.200	1.193	1.173	1.140	1.095	1.040	.976	.907	.840	.800
.21	1.210	1.203	1.182	1.148	1.102	1.045	.979	.906	.835	.790
.22	1.220	1.213	1.192	1.157	1.109	1.050	.982	.906	.831	.780
.23	1.230	1.223	1.201	1.165	1.117	1.056	.985	.907	.828	.770
.24	1.240	1.232	1.210	1.174	1.124	1.062	.989	.908	.825	.760
.25	1.250	1.242	1.220	1.182	1.131	1.068	.993	.910	.823	.750
.26	1.260	1.252	1.229	1.191	1.139	1.074	.997	.911	.821	.741
.27	1.270	1.262	1.239	1.200	1.146	1.080	1.001	.913	.820	.733
.28	1.280	1.272	1.248	1.209	1.154	1.086	1.006	.916	.819	.726
.29	1.290	1.282	1.258	1.217	1.162	1.093	1.011	.919	.819	.721
.30	1.300	1.292	1.267	1.226	1.170	1.100	1.016	.922	.820	.717
.40	1.400	1.391	1.363	1.316	1.253	1.172	1.077	.967	.845	.712
.50	1.500	1.490	1.460	1.410	1.340	1.253	1.149	1.030	.896	.750
R' / P	180°: 360 :	170°: 350 :	160°: 340 :	150°: 330 :	140°: 320 :	130°: 310 :	120°: 300 :	110°: 290 :	100°: 280 :	90° 270

Table directly applicable to HW. Change argument P by ± 90° for LW.

Table 17.-Mean height factors for diurnal tides.

R' \ P	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
0.00	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66
.01	.67	.67	.67	.67	.67	.67	.67	.66	.66	.66
.02	.67	.67	.67	.67	.67	.67	.67	.67	.66	.66
.03	.68	.68	.68	.68	.68	.67	.67	.67	.67	.66
.04	.69	.69	.69	.69	.68	.68	.67	.67	.67	.66
.05	.70	.70	.70	.69	.69	.68	.68	.67	.67	.66
.06	.71	.71	.70	.70	.69	.69	.68	.67	.67	.66
.07	.72	.72	.71	.70	.70	.69	.68	.68	.67	.66
.08	.73	.73	.72	.71	.70	.70	.69	.68	.67	.66
.09	.74	.73	.73	.72	.71	.70	.69	.68	.67	.66
.10	.74	.74	.73	.72	.71	.70	.69	.68	.67	.66
.11	.75	.75	.74	.73	.72	.71	.70	.69	.67	.66
.12	.76	.76	.75	.74	.73	.71	.70	.69	.68	.66
.13	.77	.77	.76	.75	.73	.72	.70	.69	.68	.66
.14	.78	.78	.77	.75	.74	.72	.71	.69	.68	.66
.15	.79	.79	.78	.76	.75	.73	.71	.70	.68	.66
.16	.80	.80	.78	.77	.75	.73	.72	.70	.68	.66
.17	.81	.81	.79	.78	.76	.74	.72	.70	.68	.66
.18	.82	.82	.80	.78	.76	.74	.72	.70	.68	.66
.19	.83	.83	.81	.79	.77	.75	.73	.71	.68	.66
.20	.84	.84	.82	.80	.78	.76	.73	.71	.69	.66
.21	.85	.85	.83	.81	.79	.76	.74	.71	.69	.66
.22	.86	.86	.84	.82	.79	.77	.74	.71	.69	.66
.23	.87	.87	.85	.83	.80	.77	.75	.72	.69	.66
.24	.88	.88	.86	.84	.81	.78	.75	.72	.69	.66
.25	.89	.89	.87	.85	.81	.78	.75	.72	.69	.66
R' / P	180° 360	170° 350	160° 340	150° 330	140° 320	130° 310	120° 300	110° 290	100° 280	90° 270

Take $R' = M_2/(K_1+O_1)$; P = MKO for HW factor; P = MKO±90° for LW factor.

Tabular value to be multiplied by (K_1+O_1)

Above table includes empirical corrections for disturbing effects of other constituents.

☆ U.S. GOVERNMENT PRINTING OFFICE : 1961 O—616054